ESSAYS ON OPTIMAL GOVERNMENT POLICY

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Thesis submitted for the degree of Doctor of Philosophy

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March 1993

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Acknowledgements

My greatest thanks go to my supervisors, Dr Martin Cripps, for his constant and precious help, and Prof Marcus Miller, for his devoted assistance and support.

I was also helped by Lei Zhang in numerous occasions. I benefited from the suggestions put forward by the participants at the Economic Theory Workshop of Warwick University Economics Department. Comments made by the participants at the Royal Economic Society (London, April 1992), were appreciated.

I take this opportunity to express my deepest gratitude to Dr Stefano Micossi, who has guided me since my undergraduate studies.

I am particularly thankful to Luca Papi, whose constant help and encouragement have been crucial throughout my academic life, and to my parents for their support and patience during all my years of study.

I acknowledge the financial support of the Ente per gli Studi Monetari Bancari e Finanziari 'Luigi Einaudi' and the Banco di Sicilia.

I am indebted to Monika Weber-Fahr for the figures.

Declaration

Chapter 2 is a joint work with Dr Martin Cripps. Dr Cripps helped in the solution of a technical problem that emerged in equation (36).

Summary

The aim of the thesis is to advance our understanding of the policymakers' and public's behaviour and their interactions. What emerges from the analysis is how the government chooses the optimal policy and how the public forms its expectations. Part I focuses on the time-consistency issues which arise in a managed exchange rate regime, when a country pursues exchange rate stabilization but wants to retain some monetary independence. Part II explores the strategic interactions between the government and the private sector in an environment with uncertainty and asymmetric information, in a closed economy model of monetary policy *a' la* Barro-Gordon. Part III is concerned with the same issues in an open economy model, after a change in the exchange rate regime. The techniques used involve the Bellman principle of optimality and the Kalman-Bucy filter.

The analysis of Chapter 1 points out that discrepancies between the time-consistent policy and the optimal (time-inconsistent) linear rule can emerge both in the long run equilibrium and in the path to the steady state. Chapter 2 examines how the government can take advantage of asymmetric information to achieve an initially low time-consistent inflation, although, in the long run, inflation must converge to the higher complete information equilibrium. The analysis of Chapter 3 shows how, even when the government does not attempt to manipulate the public's learning, inflation is still lower than in the steady state, immediately after the adoption of an exchange rate peg, in an open economy. This low inflation may not be costless, however, as it can be accompanied by high output losses if inflationary expectations start from a high level. Finally, Chapter 4 shows how the way in which the information asymmetry is introduced matters for the determination of government policy.

INTRODUCTION

The aim of the thesis is to advance our understanding of the behaviour of, and interactions between, policymakers and the public. The government's actions result from its objectives and the constraints it faces, hence policy decisions are endogenous. Similarly, private operators are considered to be rational and forward-looking. Thus, the strategic interaction between the policy authorities and the public constitutes the core of the thesis. What emerges from the analysis is how the government chooses the optimal policy and how the public forms its expectations. Although the entire thesis deals with optimal policy decisions, Part I studies the government/public interaction in the context of exchange rate management, and focuses on the comparison between the time-consistent and time-inconsistent strategies, while Parts II and III explore the policymakers' and private sector's behaviour in an environment with uncertainty and incomplete information. In such a realistic and complex framework, Part II attempts to explain actual and expected inflation in a closed economy, while Part III attempts to do this in an open economy after a change in the exchange rate regime.

Part I analyses optimal exchange rate management. In particular, it examines the government's optimal policy when a country aims at stabilizing the exchange rate, but wants to retain some degree of monetary independence. The attention is focused on the time-consistency issues that arise in this framework and the purpose of Chapter 1 is to build a bridge between the time-consistency literature and optimal exchange rate management studies, which seem to have neglected this important issue. Chapter 1 stems from the observation that countries participating in quasi-fixed exchange rate arrangements want to retain some monetary independence and sometimes have conflicting objectives. Therefore, the questions to be addressed here are: what is the optimal government policy? And how does the optimal time-inconsistent strategy compare with the incentive-compatible one? The analysis is carried out using a deterministic continuous-time model.

Part II builds on the well known Barro-Gordon monetary policy model in order to examine how the presence of uncertainty and asymmetric information impinges on the policy formation process. Uncertainty is a crucial element of reality and it is introduced in the form of the government's imperfect command of inflation. The introduction of information asymmetry renders the study of the strategic interactions between the policymakers and the public more interesting and complex, because the scope for manipulating the less-informed player's expectations is enhanced. Asymmetric information enters the model as lack of information on the part of the private sector about the government's preferences. The private sector's behaviour is not assumed to be gullible and naive, and their learning process reflects this. The government's optimal policy is also the optimal signalling strategy, since the policymakers take into account the information content of their actions. The analysis is carried out using a continuous-time stochastic model and the techniques used involve the Bellman principle of optimality and the Kalman-Bucy filter.

Part III, which comprises Chapters 3 and 4, is concerned with an open economy model in which asymmetric information is present.

Chapter 3 analyses the government's optimal policy, with respect to the choice of planned inflation, when a country changes exchange rate regime. The regime switch examined is that from free floating, where purchasing power parity always holds, to a pegged regime. This is a situation in which information asymmetries are particularly relevant, since the public has never before been able to observe the policymakers' preferences about competitiveness. After the regime shift, the government can affect the real exchange rate, and hence the public can gather information about the government's preferences from its actions. Such a change in the exchange rate regime is often advocated as a way of implementing a disinflation. However, some have argued that, in the presence of information asymmetries, and hence when the public has to learn about the characteristics of the new environment, the effectiveness of this anti-inflationary device could be impaired. This chapter challenges this view and aims at analysing the optimal policy and the public's evolution of inflationary expectations in such a scenario, taking fully into account the strategic interactions between the two players. The model used is stochastic and the techniques employed are the same as in Part II.

Chapter 4 explores the role of the information structure using the model developed in Chapter 3. Its purpose is to examine whether simple changes in the information structure can alter the solution to the government's optimization and, if so, to highlight the differences.

PART I

CHAPTER 1

OPTIMAL EXCHANGE RATE MANAGEMENT

1.1 Survey of the literature and introduction

Much has been written about the merits of different exchange rates regimes and on their management. During the seventies and early eighties, a large stream of studies dealt with optimal exchange rate intervention in the context of simple IS-LM stochastic discrete time models, where the government aims at minimizing output fluctuations. The general message of these studies is that, in the presence of various shocks, neither a fixed nor a flexible exchange regime is optimal; the optimal system is a compromise between the two, apart from very special cases. Furthermore, when financial disturbances are likely to be more frequent and larger than real shocks, it is desirable to have a higher degree of exchange rate fixity. On the other hand, if the real sector is more unstable, it is preferable to allow more flexibility. As Kaminow (1979) points out, this approach is a restatement of the Mundell-Fleming results concerning the effectiveness of monetary and fiscal policy with fixed and floating exchange rates. The difference lies in the fact that the studies on intervention take into consideration a continuum of regimes and adopt an optimizing approach. This voluminous literature reflects the emphasis on output and employment stabilization typical of the seventies.

Later the emphasis shifted towards the control of inflation, and as a result, exchange rate policies were directed at achieving price stability and monetary discipline. With this purpose in mind, a fixed exchange rate system, whereby a country seeking price stability links its currency to that of a country with a better monetary record, appeared to be an answer (for an extended survey on this topic see Chapter 3).

Another reason for advocating exchange rate stabilization is based on the widespread belief that exchange rate markets constitute a case of market failure

(Krugman 1989). The same normative indication is also supported by those who do not subscribe to the view of market inefficiencies, arguing that exchange rate instability stems from governments' policies (McKinnon 1988). Therefore, by the late eighties, aided by the very large fluctuations between the major currencies that occurred in that decade, consensus appeared to gather around exchange rate stabilization.

The preferred form of exchange rate stabilization was target zones rather than a fixed exchange rate system; this was in line with the theoretical results on optimal exchange rate management. Target zones do not require as much intervention as fixed exchange rate systems, and they leave some leeway for real shocks, as well as for limited demand management or some degree of monetary independence. In the context of a Dornbusch-type model, Laskar (1986) offers some rationale for this kind of exchange rate arrangement when countries aim at reducing fluctuations in output and prices. Frenkel and Golstein (1986) offer a guide to a target zone regime, providing a non-technical analysis of the reasons for having target zones and of their implementation. In addition, it was believed that target zones could alleviate the problem of speculation. The latter has been the concern of a large body of literature which dealt with the pegging of the exchange rate as a stabilization scheme for the price of a commodity (Henderson and Salant 1978; Salant 1983; Flood and Garber 1983, 1984; Krugman 1979; *et al.*). The main points made are: a) stabilization devices by means of buffer stocks are bound to fail. The failure finally occurs via a speculative attack. b) Speculative attacks are not the result of arbitrary psychological factors, but the outcome of optimizing behaviour. In such a system, the monetary authority has no degree of freedom in deciding how to intervene, since speculators will initially sell all their holdings to the government, and thereafter the government will have to sell the commodity, or foreign reserves, according to the demand induced by the fixed price. Essentially, the issue emerging from this literature is the non-sustainability of a stabilization scheme when fundamentals are against it.

Recently, following the seminal work by Krugman (1991), a new stream of studies on target zones has appeared (Miller and Weller 1991; Bertola and Caballero 1992; Svensson 1991; Flood and Garber 1992; Delgado and Dumas 1992; *et al.*; see Bertola 1991 for a technical survey). It tackles the question of the working of target zones, and in particular looks at the behaviour of the exchange rate inside the band, in the context of continuous-time stochastic models.

The literature on target zones has gone through two phases. During the first phase, target zones were portrayed as stabilization devices which displayed a stabilizing effect, even when intervention was not carried out. This is the so called "honeymoon effect" which means that, in response to a variation in fundamentals assumed to follow a Brownian motion process, the variation in the exchange rate is smaller with a target zone than with a free floating regime. (This gives rise to the famous S-shaped curve.)

In the second phase, the viability issue was addressed. The credibility and sustainability question gained momentum and a connection between price stabilization schemes and speculative attacks models was established (Krugman and Rotemberg 1992; *et al.*).

The determination of the exchange rate path also requires the specification of the intervention policy; because of the forward-looking nature of the exchange rate, it is necessary to specify the government's behaviour in order to determine exchange rate expectations. In Krugman (1991), intervention is marginal and infinitesimal; it takes place only when the exchange rate reaches the edges of the band and it is carried out just to prevent the exchange rate from crossing the boundaries. Miller and Weller (1991) also have infinitesimal marginal intervention, whilst, in their previous work, (Miller and Weller 1989) there is marginal intervention, but it is discrete so as to wipe out the interest rate differential. Also Svensson (1991) and Froot and Obstfeld (1992) deal with marginal intervention, while Flood and Garber (1992) have considered discrete infra-marginal intervention. Krugman (1991) and Miller and Weller (1991) do not offer any justification for the intervention rule considered, other than that it is exogenously given. In Flood and Garber's model, there is an infinite number of intervention rules that can sustain a certain target zone. Svensson (1991) and Froot and Obstfeld (1992) emphasize that the specification of a band for fundamentals, besides a band for the exchange rate, is necessary in order to get a unique equilibrium for the exchange rate. In all these studies, no suggestion is offered about how to choose the intervention rule. These models do not shed much light on the optimal policy to be implemented in order to sustain the target zone.

Dixit (1989) presents the optimal strategy for the control of a Brownian motion process, for a general reward function. He considers two types of costs of regulation: lump-sum costs and linear costs. Assuming the existence of a band and adopting a discrete time approximation, Dixit obtains the conditions for an optimum. Impulse control, i.e. marginal and discrete, turns out to be optimal when there are lump-sum costs. When there are only linear costs, instantaneous or barrier control, i.e. marginal and infinitesimal, is the outcome of the optimization. These results are fairly intuitive. If the costs of regulation have a fixed component, then it is better to push the process well inside the band, so that the controller will not incur these costs too often. An application of Dixit's results to the problem of exchange rate management has been carried out by Avesani (1990). Using Krugman's (1991) model, Avesani obtains results which mirror Dixit's: given that the cost function does not include any fixed component, marginal and infinitesimal intervention turns out to be the optimal strategy. Although Avesani's work is a first attempt at adopting an optimizing approach in the analysis of intervention in a target zone, several problems are not addressed, most importantly, that of time-consistency.

In the analysis of optimal exchange rate management, whilst the credibility of the band has been considered and mainly linked to the sustainability issue, the credibility of the optimal policy has not been a matter for concern. An exception in this respect is the work by Svensson (1992), which examines a managed exchange rate system without bands. However, Svensson's analysis concentrates on the optimal response to different types of shocks, while the differences between the solution under discretion and the solution under commitment are obscured.

On one hand, the time-consistency issues have found wide application to monetary policy games in a closed economy (these are surveyed in Chapter 2). On the other hand, it is very surprising to find that the studies on optimal exchange rate management have neglected the time-consistency issues which inevitably arise when agents with forward-looking behaviour are present. Some work that has been carried out along these lines, has been concerned with stabilization policy in open economies (Driffill 1982; Miller 1985; Miller and Salmon 1985; *et al.*) but not with exchange rate management.

After having surveyed the literature, the need for further research on optimal intervention rules appears clear and, in particular, the need to address the time-consistency issues in this framework. This chapter attempts to close this gap existing in the literature by setting up a model of exchange rate management. The analysis of time-consistency issues which arise in this context constitutes the core of the chapter. As these issues are more prominent in a Svensson-type framework, the analysis deals with a managed exchange rate regime without bands. In particular, the optimal policy is derived, when a country pursues exchange rate stabilization but wants to retain some degree of monetary independence. The purpose is to examine whether the optimal policy is incentive-compatible and, if not, how it compares to the time-consistent solution. Is there a difference between the two policies in the long run, or do they differ only in the convergence path towards the same long run equilibrium? This analysis attempts to build a bridge between the time-consistency literature, which is surveyed in Chapter 2, and exchange rate management studies.

The rest of this chapter is structured as follows. In Section 1.2, the model is specified, in Section 1.3 the time-consistent policy is presented, while the optimal linear rule is the subject of Section 1.4. In Section 1.5, the optimal linear, but time-inconsistent policy is compared and contrasted to the incentive-compatible policy. Section 1.6 concludes.

1.2 The model

The problem is formulated in a simple way. The government aims at minimizing a quadratic function, equation (1). In particular, policymakers want to minimize the discounted (discount factor ρ) square deviations of the exchange rate, s_t, from a fixed chosen level, $\bar{s}$, and the deviations of the money stock, m_t, from the desired value,

$\bar{m}$.

The exchange rate is determined by a standard monetary model, as shown in equation (2), where v_t is velocity and $E(ds_t/dt)$ is the expected variation of the exchange rate; expectations are assumed to be rational. Policymakers can affect the exchange rate by varying the money supply: there is a contemporaneous effect of money on the exchange rate and also an effect operating via expectations on the current and future exchange rate. The policy instrument is the rate of variation of the money supply, y_t, equation (3), while m_t is the state variable.

The choice of $\bar{s}$ is made by the policy authorities independently of the choice of $\bar{m}$. Therefore, it is possible to have conflicting exchange rate and money objectives. Two such cases can arise: the first is when policymakers want an appreciated exchange rate (low $\bar{s}$) and loose money supply (high $\bar{m}$). The second is when high $\bar{s}$ is coupled with low $\bar{m}$. The former case can happen if an appreciated exchange rate level is desirable in order to achieve low inflation, but loose money is desirable in order to boost the level of economic activity. It is also conceivable that the exchange rate target is chosen as part of an exchange rate agreement and cannot be modified unilaterally by one country, as it is the case in the Exchange Rate Mechanism (ERM) of the European Monetary System (EMS). This would imply that even if the exchange rate target was initially determined in line with the money target, subsequent changes in the money target, due for example to changes in internal economic conditions, but not in the exchange rate parity, would lead to conflicting objectives. It is possible to think of the UK before abandoning the ERM as being an example of the first case of conflicting objectives, and Germany after unification as being an example of the second case of conflicting objectives.

On intuitive grounds, it is easy to conjecture that when the two objectives are in conflict with one another, the policymakers might attempt to manipulate private agents' expectations, and in this way, given the forward-looking nature of the exchange rate, manipulate the exchange rate itself. For example, if the targets are low $\bar{s}$ and high $\bar{m}$, the monetary authority could initially reduce the money supply, hence inducing expectations of exchange rate appreciation, and then increase the money supply in order to get closer to the money stock target, whilst the exchange rate will be kept appreciated by the public's expectations. This suggests that the government could have an incentive to deviate from a chosen policy. In other words, the lack of policy instruments, compared to the number of objectives, can make the ex-ante optimal policy time-inconsistent (see Persson and Tabellini 1990).

The objective function also comprises a squared term in y_t which represents the costs of intervention. These costs could result from various sources; they could be material costs of intervention or arise from the authorities' reluctance to intervene. A squared term in y_t would also appear in the objective function, if the government aimed at minimizing variations of the domestic interest rate, in either direction. In fact, from the monetary model, assuming v_t equals zero, the interest rate is [1]:

$$i_t = -\gamma^{-1}(s_t - m_t).$$

hence, taking into account equation (5), the expected rate of variation of the exchange rate can be expressed as follows:

[1] It is assumed that the foreign interest rate equals zero, for simplicity.

$$E(di_t/dt) = = \gamma^{-1}(s_t - 1)y_t.$$

Therefore, when the government dislikes variations in domestic interest rates, the right hand side of the above equation would enter the objective function squared, i.e. this would yield a square term in y_t. The latter is the preferred justification here for the presence of intervention costs in the objective function. Since a target zone or some other form of exchange rate stabilization device introduces higher volatility in interest rates, it is reasonable to assume that the government will want to take into account both, exchange rate and interest rate variability and thus work out the best compromise between the two.

Thus the government's problem is as follows:

$$\min_{y_t} \int_0^\infty e^{-\rho t}\left[(s_t - \bar{s})^2 + (m_t - \bar{m})^2 + \frac{c}{2}y_t^2\right] dt \tag{1}$$

where

$$s_t = m_t + v_t + \gamma E(ds_t/dt) \tag{2}$$

$$\text{s.t.} \qquad \frac{dm_t}{dt} = y_t. \tag{3}$$

The problem is solved with the help of the undetermined coefficient method. The conjectural function for the exchange rate is the following:

$$s_t = s_0 + s_1 m_t + s_2 v_t, \tag{4}$$

where s_0, s_1 and s_2 are the time-invariant coefficients which have to be determined.

Without loss of generality, it is assumed, for simplicity, that v_t equals zero and thus exchange rate expectations can be expressed as:

$$E(ds_t/dt) = s_1 E(dm_t/dt). \tag{5}$$

The model is deterministic, although introducing a stochastic element by making velocity a Brownian motion would not alter the solution substantially.

The results can be summarized in the two propositions which follow.

Proposition 1

The time-consistent policy of the government, y_t^*, for the problem set up above is:

$$y_t^* = D\left[m_t - \frac{[\bar{m} + (1+\gamma s_1 D)\bar{s}]}{(2+\gamma s_1 D)}\right] \tag{6}$$

where D is

$$D = \frac{(\rho + \frac{4\gamma s_1}{c}) - \sqrt{(\rho + \frac{4\gamma s_1}{c})^2 + \frac{16}{c}(1 - \frac{2\gamma^2 s_1^2}{c})}}{2(1 - \frac{2\gamma^2 s_1^2}{c})} \tag{7}$$

and s_1 is the solution to the following polynomial

$$\frac{4\gamma^4}{c^2}s_1^6 + \frac{2\gamma^2}{c^2}[c(\rho\gamma - 2) + 2\gamma^2]s_1^4 + \frac{2\gamma^2}{c}(2-\rho\gamma)s_1^3 + \frac{1}{c}[c(1-\rho\gamma) - 4\gamma^2]s_1^2 + (\rho\gamma - 2)s_1 + 1 = 0. \tag{8}$$

In Section 1.3, proposition 1 is proved. Although it is possible to rule out solutions to the above polynomial which are not in the range (0,1) on economic grounds, it is still possible that multiple solutions arise [2].

Proposition 2

The optimal linear rule, which results in being time-inconsistent, is:

$$y_t^* = A\left[m_t - \left(\frac{\bar{s}+\bar{m}}{2}\right)\right]. \tag{9}$$

where A is the solution to the following polynomial:

$$\frac{\gamma(\gamma A^2-\rho)}{(1-\gamma A)^2(\rho-A)^2}\left[m_0(\bar{s}-\bar{m}) + \frac{1}{2}(\bar{m}^2-\bar{s}^2)\right] + \frac{(m_0-L)^2}{(\rho-2A)^2}\left[\frac{2\gamma(\rho-2A)}{(1-\gamma A)^3} + cA(\rho-A) + 2 + \frac{2}{(1-\gamma A)^2}\right] = 0 \tag{10}$$

There is a unique solution in this case. In Section 1.4, proposition 2 is proved.

1.3 The time-consistent policy

In this section, the government's problem presented above is solved using the Hamilton-Jacobi-Bellman (HJB) equation and hence dynamic programming ensures that the solution found is time-consistent. The starting point is the conjecture about government policy. It is assumed that the rate of change of money is adjusted by policymakers according to a rule which is a linear function of the state, i.e. of the money stock. The conjectural function for the policy is as follows:

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[2] Lockwood (1991) shows how multiple equilibria may be common in linear quadratic dynamic games with two players.

$$y_t = F + Dm_t, \tag{11}$$

where F and D are undetermined coefficients which are time-invariant.

The second conjecture to be made concerns the value function, V_t. The value function is assumed to be:

$$V_t\,(m_t) = e^{-\rho t}\,W_t\,(m_t) \tag{12}$$

where

$$W_t = \mu_{0t} + \mu_{1t}\,m_t + \mu_{2t}\,m_t^2\,. \tag{13}$$

The HJB equation for this problem can be written as:

$$\begin{gathered}\rho W_t - \dot{\mu}_{0t} - \dot{\mu}_{1t}\,m_t - \dot{\mu}_{2t}\,m_t^2 = \\ \min_{y_t}\{\,[m_t+\gamma s_t(F+Dm_t)-\bar{s}]^2 + (m_t-\bar{m})^2 + \frac{c}{2}y_t^2 + W_m\,y_t\}\,,\end{gathered} \tag{14}$$

where W_m is the first derivative of the function W with respect to the state variable.

The first order condition obtained by differentiation of the HJB equation is:

$$y_t^* = -\frac{(\mu_{1t} + 2\mu_{2t}\,m_t)}{c}\,. \tag{15}$$

In equation (15), the optimal control, y^*_t, is a function of the undetermined coefficients of the value function. These are determined by replacing y_t with equation (15) into the HJB equation and then equating coefficients. The equations resulting from equating the coefficients of the terms in m^2_t and m_t, are, respectively:

$$\dot{\mu}_{2t} = \frac{2}{c}\mu_{2t}^2 + \rho\mu_{2t} - 1 - (1+\gamma s_1 D)^2, \tag{16}$$

$$\dot{\mu}_{1t} = \rho\mu_{1t} + \frac{2}{c}\mu_{1t}\mu_{2t} + 2\bar{m} - 2(1+\gamma s_1 D)(\gamma s_1 F - \bar{s}) . \tag{17}$$

The solutions to equations (16) and (17) are the time-invariant solutions and the solution for μ_2 is [3]:

$$\mu_2 = \frac{c\left[-\rho + \sqrt{\rho^2 + \frac{8}{c}[1+(1+\gamma s_1 D)^2]}\right]}{4} . \tag{18}$$

From the quadratic equation for μ_2, the positive solution, shown above, was chosen, whilst the negative solution was ruled out based on the second order conditions for a minimum. Given equation (18), the solution for μ_1 is:

$$\mu_1 = \frac{2\,(1+\gamma s_1 D)(\gamma s_1 F - \bar{s}) - 2\bar{m}}{\frac{\rho}{2} + \frac{1}{2}\sqrt{\rho^2 + \frac{8}{c}[1+(1+\gamma s_1 D)^2]}} \tag{19}$$

Substitution of μ_1 and μ_2 in the control equation (15), yields an equation for the control of the form initially conjectured, so that the coefficients D and F can be determined. The equation for the control is:

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[3] The time subscripts are dropped since the solutions are constant over time. This choice is consistent with the policy conjecture.

$$y_t^* = \frac{-4\,(1+\gamma s_1 D)(\gamma s_1 F-\bar{s})+4\bar{m}}{c\rho+c\sqrt{\rho^2+\frac{8}{c}[1+(1+\gamma s_1 D)^2]}} + \tag{20}$$

$$\left[\frac{\rho}{2}-\frac{1}{2}\sqrt{\rho^2+\frac{8}{c}[1+(1+\gamma s_1 D)^2]}\right] m_t \,.$$

Combining equation (20) and equation (11) D is obtained as:

$$D = \frac{(\rho+\frac{4\gamma s_1}{c}) - \sqrt{(\rho+\frac{4\gamma s_1}{c})^2+\frac{16}{c}(1-\frac{2\gamma^2 s_1^2}{c})}}{2(1-\frac{2\gamma^2 s_1^2}{c})} \tag{21}$$

and F as:

$$F = -\frac{D\,[\bar{m} + (1+\gamma s_1 D)\bar{s}]}{(2+\gamma s_1 D)} \,. \tag{22}$$

The equation for the optimal policy is rewritten in a different way, for clarity.

$$y_t^* = D\left[m_t - \frac{[\bar{m} + (1+\gamma s_1 D)\bar{s}]}{(2+\gamma s_1 D)}\right], \tag{23}$$

where D is the negative coefficient shown in equation (21). The solution is complete once s_1 is determined. This is done by placing equation (23) into equation (5) and then (2) and combining the latter with the conjectural function for the exchange rate (equation 4). By so doing, a sixth order polynomial is obtained, viz:

$$\frac{4\gamma^4}{c^2}s_1^6 + \frac{2\gamma^2}{c^2}[c(\rho\gamma-2)+2\gamma^2]s_1^4 + \frac{2\gamma^2}{c}(2-\rho\gamma)s_1^3 + \frac{1}{c}[c(1-\rho\gamma)-4\gamma^2]s_1^2 + (\rho\gamma-2)s_1 + 1 = 0 \; . \tag{24}$$

Only the positive real roots are of interest here, since when money increases, it makes sense that the exchange rate depreciates (s_1 increases). Using Descartes' rule of signs, it is possible to say that there can either be two or four real roots. The polynomial above can be factorized into a second order and a forth order polynomial. The forth order polynomial:

$$\frac{2\gamma^2}{c}s_1^4 + (\frac{2\gamma^2}{c}+\rho\gamma-1)s_1^2 + (2-\rho\gamma)s_1 - 1 \; , \tag{25}$$

can have three or one positive real root, but there is always only one root between zero and one; this is the root chosen, whilst the others are ruled out on the basis that in a system under control the exchange rate should be stabilized (compared to the free float solution in which s_1 is unity). Furthermore, only if s_1 is comprised in the interval (0,1), D is negative; D is required to be negative in order to ensure convergence to the steady state. The solution from the second order polynomial is:

$$s_1 = \frac{1}{\gamma}\sqrt{\frac{c}{2}} \; . \tag{26}$$

This solution is only acceptable when the parameter values produce an s_1 smaller than one, for the reason mentioned above [4]. Furthermore, it has to be noted that in order to ensure that D is a real number the quantity inside the square root in equation (21) has to be positive. This imposes the following condition on s_1:

$$s_1 \leq \frac{\rho c}{4\gamma}\left(1 + \sqrt{2+\frac{16}{c\rho^2}}\right). \tag{27}$$

Since equation (26) satisfies the condition specified in equation (27), it is possible that there are two acceptable solutions for s_1 (i.e. between zero and one).

The time-consistent solution is now fully worked out and proposition 1 is proved. The resulting equilibrium is a Nash equilibrium, so that this is the optimal time-consistent policy for the government, given the private sector's strategy. The time-consistent policy is a linear rule, namely the rate of variation of money is a linear function of the money stock. Policymakers drive the money stock towards its long run level, i.e. $\dfrac{\bar{m} + (1+\gamma s_1 D)\bar{s}}{(2+\gamma s_1 D)}$, exponentially. Finally, it is worth recalling that there is the possibility that two solutions are acceptable.

[4] The limiting value of D when s_1 takes the value shown in equation (26) is finite and negative.

1.4 The optimal linear rule

In this section, the optimal linear rule is derived. Although this may not be the optimal time-inconsistent policy over all, here the search is restricted to linear rules because the rule announced should be simple. The solution procedure is as follows. First of all, the level of the money stock in the long run is calculated. In the steady state, y=0, thus it is possible simply to differentiate the objective function of the government with respect to m. This yields the following result:

$$m^* = \frac{\bar{s} + \bar{m}}{2} = L \ . \tag{28}$$

The above equation shows that, in the long run, the money stock will be set at a level which is an average (with the same weights) of the exchange rate target and the money target; this long run level is denoted by L.

In order to derive the optimal speed at which the level of the money stock shown in equation (28) is approached, the integral of the government's cost function is computed and then minimized. Before proceeding with the integration, it is necessary to specify the assumptions made in this calculation. The exchange rate is still conjectured to be a linear function of the money stock

$$s_t = \bar{s}_0 + \bar{s}_1 \, m_t \ . \tag{29}$$

Furthermore, the procedure is aimed at deriving a linear rule, in which the steady state level of the money stock is given by equation (28). Hence, the following policy function is postulated:

$$y_t = A(m_t - L) \tag{30}$$

where A is the undetermined policy coefficient which is obtained in what follows. The formula below is used in the derivation

$$\bar{s}_1 = \frac{1}{1-\gamma A} \, . \tag{31}$$

Equation (31) is easily derived by combining equation (2), (5) and (30).

The government's objective function can be integrated once the exchange rate has been replaced by equation (29). $\bar{s}_0$ can be derived from equation (29) setting s_t and m_t equal to their long run value, namely L; thus, $\bar{s}_0$ results in being

$$\bar{s}_0 = (1-\bar{s}_1)\, L \, . \tag{32}$$

$\bar{s}_0$ is then substituted by equation (32) into the government's objective function. Likewise, y_t has to be replaced by equation (30) and finally m_t is replaced by

$$m_t = L + (m_0 - L)\, e^{At} \tag{33}$$

which follows from equation (30). Having done all the necessary substitutions, the integrated cost function results in being;

$$V = \frac{1}{\rho}\left(\frac{\bar{s}^2}{2} + \frac{\bar{m}^2}{2} + \bar{s}\,\bar{m}\right) + \frac{(1-\bar{s}_1)}{(\rho-A)}\left[m_0(\bar{s}-\bar{m}) + \frac{1}{2}(\bar{m}^2-\bar{s}^2)\right] \tag{34}$$

$$\frac{(m_0-L)^2}{\rho-2A}\left(\bar{s}_1^2 + 1 + \frac{c}{2}A^2\right) .$$

The first order condition is:

$$\frac{\gamma(\gamma A^2-\rho)}{(1-\gamma A)^2(\rho-A)^2}\left[m_0(\bar{s}-\bar{m}) + \frac{1}{2}(\bar{m}^2-\bar{s}^2)\right] +$$
$$\frac{(m_0-L)^2}{(\rho-2A)^2}\left[\frac{2\gamma(\rho-2A)}{(1-\gamma A)^3} + cA(\rho-A) + 2 + \frac{2}{(1-\gamma A)^2}\right] = 0 \tag{35}$$

If a simplified case is considered, in which the exchange rate and the money target coincide, and the discount rate equals 0, then the above formula reduces to

$$1 + \frac{1}{(1-\gamma A)^2} - \frac{c}{2}A^2 = \frac{2\gamma A}{(1-\gamma A)^3}. \tag{36}$$

From the above equation, it is possible to obtain a polynomial in s^{-}_{1} which is:

$$-\frac{4\gamma^2}{c}\bar{s}_1^5 + \frac{6\gamma^2}{c}\bar{s}_1^4 + \left(\frac{2\gamma^2}{c}-1\right)\bar{s}_1^2 + 2\bar{s}_1 - 1 = 0. \tag{37}$$

It is easy to show that this polynomial has a unique solution which lies between 0 and 1 [5]. This is the solution chosen, for the reasons already pointed out in the previous section. As a result, there is a unique negative solution for A to equation (36). It can also be proven that this is the value of A which minimizes the government's objective function, by proving that the objective function is convex. In fact, letting $\bar{A}$ be the unique negative value of A which satisfies equation (36), the second derivative of the

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[5] This can be shown by computing the value of the polynomial function at $s_1^{-}=0$, which is -1, and the value of the polynomial at $s_1^{-}=1$, which is $4\gamma^2/c > 0$, and then by proving that the first derivative of the polynomial with respect to s_1^{-} is always positive in the interval between 0 and 1.

objective function at $\bar{A}$ equals:

$$\left.\frac{\partial^2 V}{\partial A^2}\right|_{A=\bar{A}} = -\frac{(m_0-L)^2}{2A}\left[\frac{2\gamma^2}{(1-\gamma A)^4}+c\right] \tag{38}$$

i.e. it is positive. Given that there is only one turning point for all negative values of A, and that the second derivative at that point is positive, the conclusion is that $\bar{A}$ minimizes the government's objective function [6]. The optimal linear rule, which has L as the long run value for the money stock, is therefore obtained. This concludes the proof of proposition 2. The policy derived here entails that the money stock follows an autoregressive process. This policy is time-inconsistent, as it does not satisfy the HJB equation. Therefore, it does not constitute an equilibrium unless a commitment strategy is available. The next section discusses how the linear rule derived in this section compares to the time-consistent policy.

1.5 A comparison between the time-consistent policy and the time-inconsistent optimal linear rule

In Section 1.3 and 1.4, the time-consistent policy and the optimal linear time-inconsistent rule have been worked out. Although both policies are linear functions of the state, so that the monetary policy drives the money stock towards its long run level, the policies differ in the convergence path towards the steady state and, more

[6] For A tending to 0 and to minus infinity, V tends to infinity.

importantly, in the steady state value of the money stock and hence also of the exchange rate.

The discrepancies in the long run solutions, which are reported below for convenience, are analysed:

$$m^{*}_{TI} = \frac{\bar{s} + \bar{m}}{2} \tag{39}$$

$$m^{*}_{TC} = \frac{(1+\gamma s_1 D)\bar{s} + \bar{m}}{(2+\gamma s_1 D)} \tag{40}$$

where the subscript TI means time-inconsistent and TC time-consistent.

First of all, it is interesting to note that when $\bar{m} = \bar{s}$, i.e. when the exchange rate target equals the money target, the long run value of the money stock is the same and it is equal to the target $\bar{s}$. In this case, in fact, there is no conflict between the two objectives; hence the government has no incentive to deviate from the optimal policy and it can hit the target.

If $\bar{m}$ does not equal $\bar{s}$, then the two equilibria are different. It is possible to see that equation (39) can be obtained from equation (40) setting D=0. This fact is used, and the knowledge that D is negative in equation (40), in order to carry out a comparison between the long run solutions with the time-consistent and time-inconsistent policies. This is done by computing the derivative of m_{TC}^{*} with respect to D which results in being:

$$\frac{\partial m^*_{TC}}{\partial D} = \frac{\gamma s_1}{(2+\gamma s_1 D)^2} \left(\bar{s} - \bar{m}\right) . \tag{41}$$

This means that when the desired level for money is higher than the exchange rate target, namely when the government pursues loose money and an appreciated exchange rate, the money stock resulting from the time-consistent policy is higher than that which would result with the optimal linear rule. The explanation for this is that the public knows the targets of the government and that, if the money stock were to be at the level obtained in steady state in the time-inconsistent case, given the public expectations of no variation in the exchange rate, there would be an incentive for the government to increase the money stock further. In this way, policymakers would get closer to the desired money level and obtain a depreciation of the exchange rate only due to the contemporaneous effect, but not through the expectations channel. Since the public knows this, it will anticipate this in its exchange rate expectations and force the government to increase the money supply more. The steady state level in the time-consistent case is such that the government has no incentive to increase money further, because the impact effect of a money increase on the exchange rate by itself offsets the advantage of approaching the money target.

On the other hand, when the exchange rate target is higher than the money target, the time-consistent policy will produce a lower level of the money stock than that which would result from the time-inconsistent linear rule. This is the specular case of that considered earlier on. If the policymakers pursue a tight monetary policy and a depreciated exchange rate, they would have an incentive to decrease the money stock if it were set at the steady state level resulting from the time-inconsistent policy, given the exchange rate expectations prevailing at that moment (i.e. of no variation in the exchange rate). This would allow the government to decrease money further in order to approach the money target, without undergoing the negative effects (in this case, this is seen as appreciation) of this policy on the exchange rate via the expectations channel. It follows that private agents anticipate this and will force policymakers to reduce money to such a level at which there would be no incentive to decrease money further, because the direct negative effects on the exchange rate of a reduction in the money stock deter any further decrease. The time-consistent solution therefore, shows a money stock which is lower than that prevailing with the optimal linear rule.

The confirmation that the time-consistent policy entails a larger loss comes from the evaluation of the value function in the steady state under the time-inconsistent and the time-consistent solution. The time-inconsistent solution yields the following value function:

$$V_{TI} = \frac{(\bar{s} + \bar{m})^2}{2\rho} \tag{42}$$

whilst the time-consistent solution yields:

$$V_{TC} = \frac{(\bar{s} + \bar{m})^2}{(2+\gamma s_1 D)\rho} \tag{43}$$

Because the loss under the time-consistent solution cannot be negative, it is bigger than that occurring with the time-inconsistent solution.

To compare the speeds of convergence towards the steady state, a simplified case is initially adopted. Namely, it is assumed that the targets for the money stock and the exchange rate coincide. The discount rate is set equal to 0, for simplicity. Under these assumptions, the speed of convergence in the time-inconsistent case, A, is the solution to the polynomial shown in equation (36), which is rewritten below for convenience.

$$1 + \frac{1}{(1-\gamma A)^2} - \frac{c}{2}A^2 = \frac{2\gamma A}{(1-\gamma A)^3} \; . \tag{44}$$

As regards the time-consistent case, only the forth order polynomial shown in equation (25) is considered, as this always provides a solution for s_1 in the (0,1) interval, whilst the solution for s_1 appearing in equation (26) is comprised in that interval only for certain parameter values. Equation (25) can be expressed in terms of D, i.e. the convergence speed in the time-consistent case, and, for ρ equal to 0, it results in the polynomial below

$$1 + \frac{1}{(1-\gamma D)^2} - \frac{c}{2}D^2 = 0 \; . \tag{45}$$

Equations (44) and (45) can now be compared. This is done by letting F(A) be

$$F(A) = 1 + \frac{1}{(1-\gamma A)^2} - \frac{c}{2}A^2 - \frac{2\gamma A}{(1-\gamma A)^3} \; , \tag{46}$$

and evaluating F(A) at $\bar{D}$, namely the speed in the time-consistent case, the function F(A) is positive. This result, together with the fact that the first derivative of F(A) with respect to A is always positive for negative values of A [7], allows the conclusion that $\bar{A}<\bar{D}$, namely that the convergence is faster in the time-inconsistent case [8].

The intuition for this result is that, when there is no discrepancy between the exchange rate and the money target, the government would get an additional bonus by announcing a rule which is faster than the time-consistent one, since this would stabilize exchange rate expectations and hence help the convergence towards the steady state, which is also the desired target for the exchange rate and the money stock. The case of no-conflict is illustrated in figure 1, which shows the stable manifolds under the time-consistent policy (the slope being s_1) and under the optimal linear rule (the slope being $\bar{s}_1$); in the figure, it is assumed that the exchange rate and the money target equal 0.

If the exchange rate and the money target differ, then the comparison between the speed of convergence to the steady state in the time-consistent and time-inconsistent case is no longer clear-cut, since the speed in the time-inconsistent case differs from the one discussed previously, i.e. when no conflict of objectives exists. If the equation defining the speed of convergence in the general time-inconsistent case is taken, and the discount rate is set equal to 0, for simplicity, the following formula attains

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[7] The limiting value of F(A) as A tends to minus infinity is minus infinity.

[8] Both $\bar{A}$ and $\bar{D}$ are negative values.

$$- \frac{\gamma^2}{(1-\gamma A)^2}\left[m_0(\bar{s} - \bar{m}) + \frac{1}{2}(\bar{m}^2 - \bar{s}^2)\right] + \qquad (47)$$

$$- \frac{(m_0-L)^2}{A^2}\left[- \frac{2\gamma A}{(1-\gamma A)^3} - \frac{c}{2}A^2 + 1 + \frac{1}{(1-\gamma A)^2}\right] = 0.$$

If $A = \bar{A}$, i.e. if the speed of convergence were to be that prevailing in the time-inconsistent case with no conflict of objectives, the second element in equation (47) would vanish. Hence, the speed when the exchange rate and money target differ is faster than when the targets coincide, and *a fortiori* faster than in the time-consistent case if:

$$(\bar{m} - \bar{s})\left[m_0 - (\frac{\bar{s} + \bar{m}}{2})\right] > 0. \qquad (48)$$

Hence, if $\bar{m} > \bar{s}$ and if the initial value of the money stock is greater than the steady state value in the time-inconsistent case, L, or if $\bar{m} < \bar{s}$ and the initial money stock is smaller than L, then one can conclude that the speed of convergence in the general time-inconsistent case is faster than in the time-inconsistent case when exchange rate and money stock objectives coincide and hence also faster than in the time-consistent case. If the quantity in equation (48) is negative, the speed of convergence is slower than in the time-consistent case [9]. The intuition for this result is that, when the exchange rate target and the money target are not consistent and hence cannot be achieved, the government will choose to announce a fast or slow (time-inconsistent)

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[9] This can be proved by studying the form of the function of A appearing in equation (47) and substituting A with $\bar{D}$ in that equation and noting that a negative quantity remains.

rule depending on which allows to keep the system closer to the exchange rate and money targets. Figure 2 and 3 illustrate the two possible conflicts, namely $\bar{s}>\bar{m}$ and $\bar{s}<\bar{m}$ respectively. The stable manifolds are drawn under the time-consistent policy (the slope being s_1) and the optimal linear rule (the slope being $\bar{s}_1$). The long run equilibria are denoted by TC and TI respectively.

1.6 Concluding remarks

The conclusion is that when the government has conflicting money and exchange rate targets, there is scope for deviations from the optimal policy even in the long run[10]. Hence, in the absence of a commitment strategy, the optimal policy will not constitute a sustainable equilibrium. The incentive-compatible solution, i.e. the time-consistent one, becomes the only viable policy, but yields a higher value for the loss function. It is also worth noting that there could be two time-consistent equilibria.

The two policies differ in both the short and the long run. In the long run, it results that, when the money target is higher (lower) than the desired level for the exchange rate, the steady state value for the money stock is higher (lower) in the time-consistent case than under the optimal linear rule.

The speed of convergence to the steady state in the time-consistent differs to that in the time-inconsistent case even when the steady states coincide, namely when no conflict of objectives exists. In the latter case, the system converges faster under the time-inconsistent rule. If the exchange rate and money objectives differ, the speed

[10] Here, the search for an optimal (time-inconsistent) policy has been limited to linear rules.

of convergence is not always faster in the time-inconsistent case.

This chapter can provide a rationale for the existence of target zones as institutional arrangements. They allow some flexibility when the government pursues more than one objective; more importantly, the institutional feature is crucial because it can offer the commitment necessary to enforce the optimal rule.

PART II

CHAPTER 2

A CONTINUOUS-TIME POLICY GAME WITH LEARNING

2.1 Survey of the literature and introduction

A positive approach to the analysis of government policy requires the specification of the government's objectives and the constraints it faces. Then it is possible to obtain the government's policy endogenously as a result of constrained optimization. The rational expectations theory revolutioned this field. By assuming that economic agents use all the available information as best as they can to avoid systematic mistakes, the environment in which the government operates is radically altered. A major contribution to this field comes from the seminal paper '*Rules rather than discretion: the inconsistency of optimal plans*' by Kydland and Prescott (1977). There it is argued that in a world where agents form rational expectations, standard optimal control theory could not be used to solve policy planning problems. In fact, optimal control would lead to the selection of a time-inconsistent plan. By this they meant that the optimal policy selected at time t for a future date, is no longer optimal once the time of the implementation arrives, even though no new information has become available. On the other hand, a policy plan is said to be time-consistent if, once the plan has been chosen, there is no incentive to deviate from it when it is due to be implemented. It has been suggested (Persson 1988) that imposing time consistency amounts to adding another constraint to economic planning, and hence, in general, the time-consistent policy yields a lower payoff.

A broad thrust of research developed from one of the examples presented by Kydland and Prescott (1977): the inflation-unemployment example which was concerned with the conduct of monetary policy. From now onwards, the attention will focus exclusively on monetary policy games. The initial static model of the natural rate offered an explanation for the inflationary bias as the time-consistent outcome of a non-cooperative game between the policymakers and the private sector (Barro and Gordon 1983a). In this framework, the time consistency issue arises because of the presence of a distortion which renders the natural level of income lower than the desired level. The same type of problem could also emerge because of the presence of externalities or because of the lack of instruments compared to the number of objectives (Persson 1988; Persson and Tabellini 1990). What occurs is that the ex-ante optimal policy results in being a second best and the government's attempts to achieve the first best (which can only be achieved by the removal of the distortion, or externality, or by the introduction of a sufficient number of instruments) leads the economy to a third best equilibrium, namely the time-consistent solution (Barro 1986a). Therefore, the optimal plan is time-inconsistent and can only be a viable solution if some precommitment technology exists. However, given the sovereign nature of every national government, it is very difficult to impose binding commitments on them; hence, a reversion to the time-consistent, or discretionary, outcome can always take place. For this reason, it is always important to examine the time-consistent policy, while it is useful to derive the optimal rule in order to have a benchmark for comparison and/or formulate normative prescriptions.

The developments of this literature enrich the framework in which the model is cast, since it makes a specific effort to model the behaviour of the policymakers and the private sector. A first step in this direction is taken by making the model dynamic (Barro and Gordon 1983b). This highlights the fact that time-consistent equilibria, where the inflation rate is a weighted average of the discretionary outcome of a one-shot game and of the optimal zero inflation rule, could be sustained in a multiperiod model. This results from reputational forces which restrain the policymakers' desire to create more inflation by means of a punishment mechanism. In this model, the problems are the multiplicity of the time-consistent equilibria and the need for an infinite horizon in order to obtain an inflationary bias lower than that present in the static game. However, both shortcomings of the Barro-Gordon model are partly overcome by subsequent studies.

The studies that followed tried to model more complex behaviour of the agents and shed more light on the strategic interactions between the players, namely between the government and the private sector. In this respect, the information structure is crucial in the determination of the government's policy and of the public's actions. In fact, the information structure affects the players' understanding of the environment they are in, and their expectations about the other player's actions. Strategic factors become particularly relevant when the information structure is asymmetric. In this case, the decisions of the more informed player --usually the government-- will also take into account the optimal way of releasing, or concealing, information to the less informed opponent --the public. Thus, in an asymmetric information framework, it is very interesting to see how the policymakers manipulate the private sector's expectations.

Backus and Driffill (1985a) and Barro (1986b) put forward a scenario where the public is uncertain about which of the two possible types of policymaker is in power, and the government's horizon is finite. In this type of model, the more inflation-prone government mimics the behaviour of the anti-inflationary government, for a while. Whilst this occurs, the public updates its beliefs about the probability that the government is anti-inflationary, using Bayes' rule. However, towards the end of the time in office the weak policymaker will inflate, producing a one-off boost in economic activity. After this, the discretionary outcome of the one-shot game attains. The equilibrium is unique and zero inflation can be sustained by reputational forces despite the finite horizon. This type of model can help explain the political business cycle and the surprises in inflation which lead to output fluctuations. On one hand, if the government in office is tough, then there will be only negative inflation surprises, on the other hand, if a weak policymaker is in power, after negative surprises, a period with a positive surprise will follow. The main drawbacks of this analysis are the presence of only two possible types of policymakers, the fact that the government only chooses between two rates of inflation and that the weak government randomizes between these two, from a certain point onwards. Also the initial value of the parameter which measures reputation is exogenously given.

In a similar setup, but where none of the two policymakers is committed to zero inflation, Vickers (1986) finds that the 'dry' policymaker, namely the one more concerned about inflation, can decide to signal its preferences by producing very low inflation. However, multiple equilibria are possible. Some are separating, as the one described above, while some others are pooling. In the pooling equilibria, the 'wet' type manages to masquerade as 'dry', but the true preferences of the incumbent policymaker are revealed in the final period of the game.

Another example of the dramatic effects of changes in the information structure is provided by Canzoneri's paper (1985) which leads to a re-appraisal of the normative conclusion of Rogoff (Rogoff 1985). Rogoff, adopting a framework germane to that of Canzoneri but with symmetric information, argues that in the presence of shocks that can be offset by government policy, a fixed rule yielding zero inflation is no longer optimal. In fact, the discretionary outcome would have the disadvantage of higher inflation but the advantage of output stabilization, so that a zero inflation rule cannot unambiguously be said to yield a higher payoff than the time-consistent solution. Rogoff therefore concludes that credibility and stabilization problems cannot be considered separately and there exists an optimal degree of commitment. In Canzoneri's study, the asymmetric information comes from the fact that the monetary authorities have private information about their forecast of a shock affecting the money demand function. This simple change produces reversionary periods, i.e. periods in which the public punishes the government by setting high inflationary expectations, even if the policymakers have not cheated and have only attempted to offset shocks. This highlights the fact that the private sector's inability to monitor the authorities' actions closely can render the working of reputational forces more shaky, so that high inflation might still recur. With this quite realistic information structure, the model can account for variations in actual and expected inflation, besides allowing that reputation can be lost but subsequently regained. However, this model requires an infinite horizon in order to yield these results.

Cuckierman and Meltzer (1986) introduce uncertainty on behalf of the public about the government's changing preferences. In this study, the policymakers' manipulation of the private sector's expectations comes out very clearly. The authorities decide to control money growth more noisily than the minimum possible given the available technology. This strategy leads to a reduction in the public's monitoring and therefore the production of positive (negative) inflation surprises when these are more advantageous (disadvantageous). It is also worthwhile noticing that the optimal degree of ambiguity is chosen by the government.

In contrast to Cuckierman and Meltzer's analysis of the steady state, Cripps

(1991) examines the convergence path towards the steady state in a model where the public does not know the government's preferences. Cripps considers two types of learning, rational and non-rational. In the case of rational learning, the presence of asymmetric information leads to a reduction of the inflationary bias, but in the long run the solution converges to the complete information outcome. When non-rational learning takes place, even the long run solution is affected and the reduction in inflation is permanent. Cripps' analysis is conducted using a discrete-time model.

The literature surveyed above advances the theory of optimal policy planning and goes a long way from the original static framework in modelling complex behaviour on behalf of policymakers and the public alike. The models are extended to include information asymmetry, a very realistic assumption which yields considerable modifications to the players' strategies. Nevertheless, there is still need to understand more about the strategic interactions between policymakers and private agents in such a setup.

This chapter therefore attempts to elaborate a model in which the private sector learns about the government's preferences. The public learns about a parameter of the policymakers' objective function; this is equivalent to assuming that the possible types of policymakers are infinite. The analysis is focused on the convergence towards the steady state. This work generalises Cripps (1991), providing an explicit calculation for the time path of government policy when it is attempting to acquire a reputation. The main difference with respect to Cripps' work is the fact that the analysis is conducted in continuous time. The techniques used are considerably less cumbersome than discrete-time ones and could, therefore, be fruitfully employed to solve more complex models; an example of this is provided by the model developed in the next chapter.

In the model below, the monetary authorities and the public sector engage in a repeated game in which the government tries to acquire a reputation for being tough and the public tries to learn its type. Using a Kalman-Bucy filter, the private sector's mechanism for updating expectations is derived, and the government's optimal time-consistent policy is obtained using the Bellman principle of optimality. The policymakers' actions provide information used by the public in the revision of their expectations. The authorities' actions are hence the endogenous source of information in the model, and since they are aware of this, the optimal policy is also the optimal signalling strategy.

The rest of the present chapter is organized as follows. In Section 2.2 the model is introduced, the details of the information structure illustrated and the solution procedure sketched. In Section 2.3, the Kalman-Bucy filter is employed to solve the private sector's learning problem. The optimal policy of the government is worked out in Section 2.4. Section 2.5 concludes.

2.2 The model

The model presented here shares with a standard policy game *a' la* Barro-Gordon the objectives of the government which likes to create surprises in inflation, as these boost output, but dislikes inflation *per se*. The government's objective function is specified below:

$$E_{gov}\left[\int_0^\infty e^{-\rho t}\left[(c+u)(\pi_t-\pi_t^e) - \frac{1}{2}(\pi_t^p)^2\right] dt\right], \tag{1}$$

where:

E_{gov} = expectation operator with respect to the government's information set;

ρ = discount rate;

c + u = weight attached to unexpected inflation;

π_t = actual inflation rate at time t;

π_t^p = planned inflation rate at time t;

$\pi_t^e = E(\pi_t)$ = expected inflation where the superscript e denotes the expectation taken with respect to the private sector's information set, E.

The time horizon is assumed to be infinite.

At this point, it is worth pointing out that, under symmetric information, the solution to the problem described above is the following:

$$\pi_t^p = c + u. \tag{2}$$

The equation above shows an inflationary bias equal to that present in a one-shot game.

The model is then enriched by its stochastic feature, and by the dynamics that result from the introduction of the asymmetric information structure. Uncertainty arises from the government's imperfect control of inflation, due to the fact that the policymakers' actions affect the inflation rate only indirectly. For simplicity, the government's choice variable is called planned inflation. The imperfect control feature is modelled by setting actual inflation equal to planned inflation plus a white noise disturbance, i.e.

$$\pi_t = \pi_t^p + \eta_t \, . \tag{3}$$

It follows that the actual price level, p_t, evolves according to the following stochastic differential equation

$$dp_t = \pi_t^p \, dt + dz_t \, , \tag{4}$$

where z is a standard Brownian motion process.

The asymmetric information comes from the fact that the public does not know the relative weight of the two terms present in the government's objective function, namely it does not discern how much policymakers care about output, and hence how much they would like to create inflation surprises, compared to the objective of keeping inflation down. It is assumed that while the average propensity to create inflation surprises is known to the public, i.e. c is common knowledge, u is not known to the private sector. The parameter u is drawn by the policymakers from a normal distribution with mean 0 and variance σ_u^2, at the beginning of the game, once and for all. This is tantamount to assuming that the possible types of policymakers are infinite. Private agents cannot observe the outcome of the drawing: thus the public only knows the average propensity of the government to create inflation. Nevertheless, the public is aware of the characteristics of the distribution from which u has been drawn. The mean of the distribution is the public's prior, whilst the variance of the distribution is a measure of the dispersion of the beliefs of the private sector about the policymakers' preferences. The private sector will have to learn the value of this preference parameter by observing the actions of the government over time. One further element of the information structure is constituted by the assumption of the policymakers' private information concerning their choice variable. This means that the policy authorities do not reveal the planned inflation rate even ex post. The implication of this is that the public cannot monitor the government's actions precisely and its source of information is actual inflation. Thus, the public's expectation operator, E, refers to an information set which is a subset of the authorities', whose expectation operator is E_{gov}.

The solution procedure is now illustrated. From the observation of actual inflation, the private sector gathers information, using an optimal Kalman-Bucy filter procedure, about the unknown element of the policymakers' preferences, and hence revises its strategy, namely the expected inflation rate, optimally. The government is aware of the private agents' learning process and the set up of the model presented here allows the authorities to solve their optimization problem taking it into account. The two players engage in a non-cooperative dynamic game of asymmetric information, in which both act strategically. These interactions constitute the core of the analysis; the aim is to analyze how the public's inflationary expectations and the government's choice of planned inflation are affected by the asymmetric information structure and the stochastic environment.

The solution consists of the optimal strategy of the two players. On one hand, private operators have to form the best estimate of the unknown parameter, on which the inflationary bias depends, so as to minimize forecasting errors; this is their objective function. On the other hand, the policy authorities have to choose the optimal strategy for the inflation path, given their preferences and taking into account the informational content of their actions [1]. The public's filtering problem and the government's optimization can be solved in a sequence of stages because the separation principle applies; it applies given the linearity of the stochastic differential equation, equation (10) below, and the fact that the objective function is quadratic in the control, π_t^p, and linear in the state variable, u_t^*, (see Davis 1977).

The solution procedure begins by postulating the government's optimal policy. It is conjectured to be linear in (c + u), i.e. it is assumed to be

$$\pi_t^p = (c+u)\ A_t\ , \tag{5}$$

where A_t is a fixed undetermined coefficient which will be optimally chosen when the government's optimal strategy has been solved for. A_t is assumed to be common knowledge. The conjectured policy function is then used in the filtering procedure which yields the optimal estimate of the parameter u, i.e. u^*_t, given the information available to the private sector. The stochastic differential equation which describes the public's updating of beliefs becomes part of the policymakers' optimization. In fact u^*_t becomes the state variable in this problem. The Bellman principle of optimality is employed in the government's optimization. The outcome is a solution to the problem at stake, only if the resulting policy is of the form initially postulated, so that it is possible to solve for the undetermined coefficients of the conjectural policy function.

The resulting equilibrium is a rational expectations equilibrium and the equilibrium is perfect bayesian.

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[1] The strategy space which comprises all admissible strategies is the same for both players and is the real line.

2.3 The public's learning: the Kalman-Bucy filter

The public's strategy is given by the expected inflation rate, which is:

$$\pi_t^e = (c+\hat{u}_t)\, A_t \ . \tag{6}$$

Private agents select their choice variable such that it minimizes their objective function, which is:

$$J = E\left[\int_0^\infty (\pi_t - \pi_t^e)^2\, dt\right] . \tag{7}$$

The above equation expresses the fact that the private sector aims at minimizing the mean square error of the forecast of the inflation rate; this, given the information structure assumed here, is equivalent to minimizing the mean square error of the forecast of the parameter u.

The public's learning problem can be formalized as follows:

$$du = 0 \ , \qquad E(u_0) = 0 \quad E(u_0^2) = \sigma_u^2 \ , \tag{8}$$

$$dp_t = \pi_t^e\, dt + dz_t = (c+u)A_t\, dt + dz_t \ . \tag{9}$$

Equation (8) represents the dynamics of the system, which being a parameter in this case, does not vary over time. Equation (9) describes the dynamics of the observations, that is the actual inflation rate. Since both equations are linear, the estimate of u, can be obtained from a Kalman-Bucy filter procedure (see Oksendal, 1985). The filter, u^*_t, satisfies the stochastic differential equation below:

$$d\hat{u}_t = \frac{A_t}{K_t}dp_t - \frac{cA_t^2}{K_t}dt - \frac{A_t^2\,\hat{u}_t}{K_t}dt\,, \tag{10}$$

where $K_t = \int_0^t A_s^2 ds + 1/\sigma_u^2$ is the inverse of the solution, S_t, to the Riccati equation

shown below:

$$\frac{dS_t}{dt} = -A_t^2 S_t^2 \qquad S_0 = E\left[u_0 - E(u_0)\right]^2 = \sigma_u^2\,. \tag{11}$$

The solution to the Riccati equation, S_t, provides the mean square error of the estimate of u, at time t. Equation (10) describes the evolution of the estimate of u, i.e. the public's updating of expectations. It shows that the speed of the private sector learning depends on various factors. One is the discrepancy between actual and expected inflation; the greater the discrepancy, the higher the speed of expectations updating. This entails that the learning is faster at the beginning of the game and slows down as private operators become more accurate in their estimate. Another is the variance of the distribution of u, i.e. σ^2_u; the larger the degree of the public's uncertainty, the faster the learning. Finally, the policy coefficient, A_t , impinges on the updating of expectations; this implies that the government can decide to slow the learning, by choosing A_t smaller than 1, or speed up the learning, by choosing A_t

greater than 1, or leave it unaffected, if A_t equals 1 [2].

Ito's Lemma is now applied to the stochastic differential equation for the filter in order to obtain the solution, i.e. the public's estimate of u. First, equation (10) is rearranged in the following way:

$$K_t d\hat{u}_t + A_t^2 \hat{u}_t dt = A_t dp_t - cA_t^2 dt \ . \tag{12}$$

Ito's lemma is then applied to a function of the form:

$$g_t = K_t \hat{u}_t \tag{13}$$

so that

$$dg_t = d(\hat{u}_t K_t) = K_t d\hat{u}_t + A_t^2 \hat{u}_t dt \ . \tag{14}$$

Equating equation (12) and equation (14), the following equation is obtained:

$$d(\hat{u}_t K_t) = A_t dp_t - cA_t^2 dt \tag{15}$$

and integrating both sides, it results in being:

$$\hat{u}_t K_t = \int_0^t A_s dp_s - \int_0^t cA_s^2 ds + R \ , \tag{16}$$

where R is an arbitrary constant. Now recalling the expression for K_t, and substituting

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[2] The sequence of A_ts affects also K_t; the larger A_t is, the larger K_t. However, if one considers only one A_t at a time, keeping the others fixed, the effect of K_t is infinitesimal, whereas there is obviously a finite effect on A_t and hence the latter prevails.

into equation (16), u_t^* is derived as:

$$\hat{u}_t = \frac{\int_0^t A_s dp_s - \int_0^t cA_s^2 ds + R}{\int_0^t A_s^2 ds + 1/\sigma_u^2} \tag{17}$$

The arbitrary constant R can be determined evaluating the previous equation at time 0. It follows that:

$$R = \frac{\hat{u}_0}{\sigma_u^2} . \tag{18}$$

Since $u^*_0=0$, $R=0$. Thus the solution to the filtering problem is the following:

$$\hat{u}_t = \frac{\int_0^t A_s dp_s - \int_0^t cA_s^2 ds}{\int_0^t A_s^2 ds + 1/\sigma_u^2} . \tag{19}$$

This is the best estimate based on the observations available to private operators. It is best in the sense that it is the estimate which minimizes the expected value of the integral of the square deviations of the estimate from the true value of the parameter, measurable with respect to the σ-algebra generated by the observations up to the time when the estimate is computed. It is worthwhile noticing that at time zero the public's estimate of the parameter is the prior, namely the mean of the distribution from which u has been drawn. As time tends to infinity, the estimate converges to the actual value of the parameter. This implies that private agents will learn the government's preferences.

However, the public's learning is not fully worked out until the policy coefficient A_t , is determined in the government's optimization. The policymakers' problem is solved in the next section.

2.4 The government's optimization

The government maximizes its objective function already illustrated, subject to the constraint describing the evolution of the public's estimate of the preference parameter, u. The policy authorities' optimization problem can now be stated as follows,

$$\max_{\pi_t^p} E_{gov} \left[\int_0^\infty e^{-\rho t} \left[(c+u)(\pi_t - \pi_t^e) - \frac{1}{2}(\pi_t^p)^2 \right] dt \right], \tag{20}$$

$$\text{s.t.} \qquad d\hat{u}_t = \frac{A_t}{K_t}\left(\pi_t^p - cA_t - A_t\hat{u}_t\right) dt + \frac{A_t}{K_t} dz_t , \tag{21}$$

where $\pi_t^e = E(\pi_t) = (c+\hat{u}_t)A_t$. The control variable chosen by the government is π_t^p, and the state variable is $\hat{u}_t$. The state variable is introduced in this asymmetric information setup because the government takes into account the effect of its actions on the public's learning process, when carrying out the maximization. Equation (21) results from the filter of the previous section.

The procedure for the solution of this stochastic optimal control problem involves the use of Bellman's principle of optimality and the application of the method of undetermined coefficients. From this, the government's optimal policy for planned inflation will result; this strategy will be measurable with respect to the policymakers' information set which includes all the history of all the variables of the model except the increment of the Brownian motion, dz_t. The main finding of this exercise is that the government's optimal policy involves an inflationary bias lower than that prevailing under complete information. However, in the long run, the private sector learns the true preferences of the policymakers and hence the government's policy converges to that of the symmetric information scenario.

Proposition

The equilibrium strategy of the government, $(\pi_t^P)^*$, obtained from the game specified above is:

$$(\pi_t^P)^* = (c+u)\left(1 + A_t e^{\rho t}\left[\int_0^t \frac{A_s e^{-\rho s}}{K_s}\,ds - \bar{A}\right]\right). \tag{22}$$

where

$$\bar{A} = \lim_{t\to\infty}\int_0^t \frac{A_s e^{-\rho s}}{K_s}\,ds . \tag{23}$$

and A satisfies the following differential equation:

$$A_t\ddot{A}_t - 2(\dot{A}_t)^2 - \rho A_t^2\dot{A}_t + \rho^2 A_t^2 (A_t - 1)^2 = 0 . \tag{24}$$

Proof

The first step of the solution involves an assumption about the functional form of the value function. It is postulated to be linear in the state variable, as shown below:

$$V_t(\hat{\theta}_t) = e^{-\rho t}\, W_t(\hat{\theta}_t) \tag{25}$$

where

$$W_t = \mu_{0t} + \mu_{1t}\,\hat{u}_t \tag{26}$$

and μ_{0t} and μ_{1t} are undetermined coefficients. At this stage the Hamilton-Jacobi-Bellman (HJB) equation can be written down, it results in

$$\rho W_t - \dot{\mu}_{0t} - \dot{\mu}_{1t}\,\hat{u}_t = \max_{\pi_t^p}[(c+u)(\pi_t^p - cA_t - A_t\hat{u}_t) - \frac{1}{2}(\pi_t^p)^2 + \mu_{1t}\frac{A_t}{K_t}(\pi_t^p - cA_t - A_t\hat{u}_t)]\ . \tag{27}$$

A dot denotes the first derivative with respect to time. The first order condition is subsequently derived by differentiation of the Bellman equation with respect to the control variable. The control is:

$$\pi_t^p = (c+u) + \mu_{1t}\frac{A_t}{K_t}\ . \tag{28}$$

The next stage of the solution procedure involves the determination of the undetermined coefficient of the value function, namely μ_{0t} and μ_{1t}. This can be done by substituting the control, π^p_{t}, with the right hand side of equation (28) in the HJB equation. In this way an identity is obtained; therefore, it is possible to equate the coefficients of the terms in u^*_t and the terms depending only on time, on both sides of the Bellman equation. This yields two differential equations for the coefficients of the value function, μ_{0t} and μ_{1t}, which are:

$$\dot{\mu}_{0t} = \rho\mu_{0t} + cA_t\left(c + u + \mu_{1t}\frac{A_t}{K_t}\right) - \frac{1}{2}\left(c + u + \mu_{1t}\frac{A_t}{K_t}\right)^2 \tag{29}$$

$$\dot{\mu}_{1t} = \left(\rho + \frac{A_t^2}{K_t}\right) \mu_{1t} + (c+u)A_t \ . \tag{30}$$

The solution to equation (30) is fully developed in Appendix A, while only the solution is reported here, as it is necessary for the derivation of the optimal policy. It is given by

$$\mu_{1t} = (c+u)e^{\rho t}\ K_t \left[\int_0^t \frac{A_s e^{-\rho s}}{K_s}\, ds - \bar{A}\right] \tag{31}$$

where

$$\bar{A} = \lim_{t\to\infty} \int_0^t \frac{A_s e^{-\rho s}}{K_s}\, ds \ . \tag{32}$$

$\bar{A}$ has been chosen as the arbitrary constant in the differential equation for μ_{1t} because it is the only constant which yields a finite value of μ_{1t} [3]. The term μ_{1t} is always negative and its limiting value as time goes to infinity is

$$\lim_{t\to\infty} \mu_{1t} = -\frac{(c+u)A_t}{\rho} \ . \tag{33}$$

It is now possible to substitute the explicit solution for μ_{1t} in the equation for the control, namely equation (28), and hence obtain

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[3] Notice that $\bar{A}$ is finite since the integrand tends to 0 as time goes to infinity.

$$\pi_t^p = (c+u)\left(1 + A_t e^{\rho t}\left[\int_0^t \frac{A_s e^{-\rho s}}{K_s}\, ds - \bar{A}\right]\right). \tag{34}$$

The solution is complete only once the sequence of A_t's over time has been worked out. This requires the application of the method of undetermined coefficients, so to obtain A_t ,

$$A_t = 1 + A_t e^{\rho t}\left[\int_0^t \frac{A_s e^{-\rho s}}{K_s}\, ds - \bar{A}\right]. \tag{35}$$

From the above equation, an important result can already be noted, that is, as time tends to infinity, A_t tends to 1, and hence the control becomes simply the symmetric information solution. This means that in the long run, as the public learns the government preferences, the policymakers are forced to revert to the complete information scenario. It can be concluded that the information asymmetry does not affect the steady state.

Equation (35) is then rearranged and differentiated with respect to time twice, to obtain the following second order differential equation for A_t :

$$A_t \ddot{A}_t - 2(\dot{A}_t)^2 - \rho A_t^2 \dot{A}_t + \rho^2 A_t^2 (A_t - 1)^2 = 0, \tag{36}$$

where two dots denote the second derivative with respect to time. The proof in now completed. The only stationary equilibria occur at A_t=0 and A_t=1. The above second order differential equation is then reduced to a system of two first order equations.

Letting $\dot{A}_t = Y_t$, and dropping the time subscripts, the following system results

$$\dot{A} = Y \tag{37}$$

$$\dot{Y} = \frac{1}{A}[2Y^2 + \rho A^2 Y - \rho^2 A^2 (A-1)^2] . \tag{38}$$

The phase diagram reported in figure 4 represents the above system. On the horizontal axis, there is A_t, while on the vertical axis there is the first derivative of A, Y_t. The horizontal axis is also the stationary locus for A. The other two lines in the picture represent the stationary loci for the variable Y, i.e. the second time derivative of A. A stationary equilibrium occurs when the stationary locus for A and for Y cross; this takes place at 0 and 1, as already pointed out above.

In order to study the behaviour of the system around the equilibria, it is linearized around the stationary point $A_t=1$ (which will turn out to be the only relevant one). The system is non-hyperbolic, as one of the eigenvalues is 0. Hence, in order to study the stability of the system, it is necessary to use the centre manifold theorem. It is possible to show (see Arrowsmith and Place, 1990, p.99) that the centre manifold goes through the stationary point $A_t=1$, and, provided $A_t<1$, the system will converge to this equilibrium along the centre manifold, in a monotonic fashion.

It is now necessary to determine the initial value for A_t, namely A_0. This is done by evaluating equation (35) at time 0. It results that,

$$A_0 = \frac{1}{(1+A)} \ . \tag{39}$$

Hence, since the limit of A_t is finite, A_0 cannot be 0. On the other hand, it can only be 1 if σ_u^2=0, i.e. if there is no uncertainty. It is thus possible to conclude that, in the presence of uncertainty, A_0 must lie in the interval (0,1). This implies that the only relevant stationary point is A_t=1, as anticipated, and it has already been said that the system will converge to it. Furthermore, the initial value of A_t depends on the discount rate and on the variance of the distribution from which u is drawn. The larger ρ, the closer A_0 is to 1. Whereas, the larger σ_u^2, the closer A_0 is to 0.

The solution therefore, says that given the public's lack of information and learning process, the government can sustain lower time-consistent inflation rates than that prevailing in the symmetric information framework. The initial inflation rate depends on ρ and σ_u^2, namely on the policy authorities' discount rate and the dispersion of the distribution of p (which can be interpreted as the private sector's degree of uncertainty), and then the initial inflation rate increases monotonically over time tending to the full information solution. Hence, the incomplete information assumption has important consequences in the transition phase towards the steady state. The long run, however, is not affected by the information structure.

As pointed out above, the value of A_t impinges on the public's learning process; having found that it is lower than 1 signifies that the government's optimal strategy entails slowing down the revision of expectations. By producing lower inflation rates, the policymakers render the relative size of the noise, present in the observations, bigger, and thus the informational content of the actual inflation rate is reduced.

Finally, the results obtained mirror Cripps (1991), however the procedure employed here turns out to be considerably less cumbersome and the results much clearer. Hence, it appears that these techniques could be used in order to tackle other models of strategic interaction between two players, where both behave optimally and one learns from the other's actions.

2.5 Concluding remarks

Continuous-time stochastic techniques, namely the Kalman-Bucy filter and the Bellman principle of optimality, are used to solve a monetary policy game model with asymmetric information, where learning takes place. In the presence of uncertainty on the part of public about the government propensity to create inflation surprises, the government chooses a time-consistent inflation which is lower than that prevailing under symmetric information. Furthermore, the public's learning is slowed down by the government. However, in the long run, the solution obtained under asymmetric information converges to the outcome resulting under symmetric information. This entails that the policymakers cannot conceal their preferences for ever. It turns out that continuous-time techniques are considerably less cumbersome than discrete-time ones and provide clearer results. Hence, there seems to be scope for their application in solving a wide class of similar economic models.

Appendix A

In this appendix, the solution to the differential equation (30) for μ_{1t} is worked out. For convenience, the equation is rewritten below, rearranged in the following way:

$$\mu_{1t} - \left(\rho + \frac{A_t^2}{K_t}\right)\mu_{1t} = (c+u)A_t \ . \tag{40}$$

Recalling the expression for K_t, one can see that

$$\frac{d\log K_t}{dt} = \frac{A_t^2}{K_t} \ . \tag{41}$$

Therefore, the above equation can be solved as a standard first order linear differential equation with time-varying coefficients. The solution is:

$$\mu_{1t} = (c+u)e^{\rho t} K_t \left[\int_0^t \frac{A_s e^{-\rho s}}{K_s} ds + \bar{A}\right] . \tag{42}$$

where $\bar{A}$ is an arbitrary constant which is

$$\bar{A} = \lim_{t\to\infty} \int_0^t \frac{A_s e^{-\rho s}}{K_s} ds \ . \tag{43}$$

This is the only arbitrary constant which ensures that the value of μ_{1t} is finite at any time. This limit is finite because the value of the integrand tends to 0, as time tends to infinity.

The limiting value for μ_{1t}, as time tends to infinity, can be computed applying De l'Hopital rule and it results in:

$$\lim_{t \to \infty} \mu_{1t} = -\frac{(c+u)A_t}{\rho}. \tag{44}$$

PART III

CHAPTER 3

HOW DOES LEARNING AFFECT INFLATION AFTER A SHIFT IN THE EXCHANGE RATE REGIME?

3.1 Survey of the literature and introduction

A pegged exchange rate system, like the Exchange Rate Mechanism (ERM) of the European Monetary System (EMS), is often portrayed as an anti-inflationary device. Taking the ERM as an example, nominal convergence, widely understood as convergence in cost and price performance towards the lowest inflation rate (Ungerer *et al.* 1986), has been one of the main priorities of the system since its inception. Many argue that the core of the EMS, i.e. the ERM, is intended to help in the pursuit of price stability and has in fact significantly contributed to the disinflation of participating countries in the eighties.

The theoretical literature concerned with the channels through which an exchange arrangement can affect price performance can be grouped in two categories[1]. The first is labelled the *disinflation literature*. Taking high inflation as given, it examines the effects of an exchange rate rule on the disinflation process and on the related output costs. The second involves a *game theoretic approach* which attempts to explain the causes of high inflation equilibria. The issue here is to see how the pegging of the exchange rate could modify the equilibrium or time-consistent inflation rate.

In the disinflation literature the attention is focused on the sacrifice ratio, i.e., the output costs of disinflation. In order to analyse this problem, Fischer used a rational expectations model with staggered contracts. Since in an open economy under the assumption that purchasing power parity (PPP) always holds, the effects of a

[1] The empirical evidence, which will not be reviewed here, concerning the ERM's contribution to the disinflation of the eighties is mixed (Christensen 1987; Dornbusch 1989; De Grauwe 1989a and 1989b; Gressani *et al.* 1988; Giavazzi and Giovannini 1989; Kremers 1990; Ungerer *et al.* 1986; Weber 1991; *et al.*).

disinflation policy are the same as in a closed economy, Fischer's closed economy model is considered first (Fischer 1984). Assume that the system is initially in steady state with constant monetary growth and a constant inflation rate. If an unanticipated contractionary policy is implemented in order to reduce inflation, the outcome will be a reduction in output, even if the policy enjoys full credibility [2]. The reason is that prices decline by less than the money stock because wages are not immediately renegotiated and thus real money balances fall. If the private sector does not believe the policymakers' programme and interprets the contraction in money growth as a reduction in the level of the money stock, then output will remain below its equilibrium level. If agents learn, and adjust their expectations consistently, then output will eventually return to its equilibrium level: however, the sacrifice ratio is greater than in the case of full credibility.

The issue at stake is how the sacrifice ratio would be affected by the introduction of an exchange rate rule like the ERM. Fischer utilizes a Dornbusch model with overshooting, so that a contractionary monetary policy leads to an appreciation of the real exchange rate (Fischer 1988). The latter is a consequence of the staggered labour contracts and will be more conspicuous the longer it takes for all contracts to be renegotiated. Although Fischer's study does not consider an ERM-type of regime, it can be a suitable framework for this purpose since, for a high inflation country, fixing the nominal exchange rate would similarly bring about a real appreciation and therefore have the same consequences with regard to disinflation.

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[2] If the anti-inflationary policy was anticipated, so that between the announcement and the implementation all contracts are renegotiated, then there would be no output loss. However, this entails putting up with the existing inflation rate for a longer time. If the policy is anticipated but wages cannot be renegotiated before the implementation, the effects are the same as in the case of an unanticipated programme.

When the exchange rate appreciates, there is an immediate reduction in the CPI because the price of imported final goods decreases. By the same token, there is a fall in the imported input costs; this increases aggregate supply and thus brings down the price of domestic goods. Moreover, a fall in wages will occur since they respond to prices. All these effects reduce the sacrifice ratio, as they produce an immediate downward push on prices. However, there is the contractionary effect on output exerted by the real appreciation; this makes the sacrifice ratio greater compared to the benchmark case, i.e., free floating when the exchange rate is at its PPP value all the time. Therefore, in this model, the sacrifice ratio depends on the parameters which determine the sensitivity of output to the real exchange rate and to the real interest rate, on the sensitivity of prices to the real exchange rate, as well as on the responsiveness of wages to both output and prices. Thus, comparison of the sacrifice ratio when the real exchange rate is kept constant to a case with an ERM-type exchange rate arrangement does not yield clear-cut results; without knowing the parameters values, one cannot tell whether the ERM has modified the output inflation trade-off faced by the policymakers.

Although the comparison of an ERM-type regime to a PPP model is a standard one in the literature, the policy-oriented debate places greater emphasis on the alternative option of implementing a disinflation program with a contractionary monetary policy. Differential effects of disinflation when pursued, alternatively, via monetary restraint and via fixing the exchange rate can be appreciated using van der Ploeg's two-country model (van der Ploeg 1986). Here, if one compares the sacrifice ratio under an ERM-type regime to the sacrifice ratio in the case of a contractionary monetary policy carried out by one country, the crucial element becomes the degree of overshooting. On one hand, fixing the nominal exchange rate would entail a deeper output loss, because it has no direct effect on core inflation and the downward push on prices occurs only through real exchange rate appreciation. On the other hand, with monetary contraction, if the overshooting is very large, it could offset the beneficial direct effect of the reduction of core inflation. As regards inflation performance, it is possible that a contractionary monetary policy combined with a free floating regime might induce a stronger squeeze on prices than a ERM-type regime, as the real exchange rate could appreciate even more. The critics of the ERM point out that during the disinflation of the eighties, countries outside the ERM have been able to adopt more drastic strategies and hence obtain a more rapid decline of inflation (De Grauwe 1989a, 1989b). However, a large appreciation might become politically unacceptable and encounter other kinds of constraints.

One reason why the use of the exchange rate is recommended as nominal anchor in a disinflation programme is that the price of foreign currency is an easily monitored key-price that can perform a signalling role (Dornbusch 1986; Bruno 1990). Similarly, an argument often advanced to support the thesis of the ERM as anti-inflationary is that membership provides a "credibility bonus" to high-inflation countries. The importance of credibility in assessing the costs of disinflation is already clear in Fischer's analysis. The output loss of a disinflation policy is a decreasing function of the credibility attached to the policy.

Even more compelling is Blanchard's argument that different expectations can give rise to different equilibria; hence success or failure of a policy might crucially depend on its credibility (Blanchard 1985). As credibility is so important, in a scenario where the public is uncertain about the policymakers' preferences, there is scope for deviations from the strategy that would be optimal in the case of symmetric information. In particular, a "conservative" government might choose an exchange rate level more appreciated than optimal, in order to establish credibility; the exchange rate is particularly suitable for this purpose because it is easily observable without lags (Winkler 1991). Although the exchange rate constraint for the countries participating in the ERM poses a limit to the degree of appreciation, the fact that it is a multilateral agreement could be a compensating advantage. The use of monetary policy, rather than the fixing of the exchange rate, can be appropriate when the aim is the reduction of inflation, not just in one country, but in the whole area (van der Ploeg 1986).

The game theoretic approach explains why a high inflation equilibrium might materialize. This literature, concerning monetary policy games in a closed economy, has been reviewed in the previous chapter. The inflation rate turns out to depend on: (i) the policymakers' preferences with respect to inflation and output --the stronger the dislike of inflation, the lower the equilibrium inflation rate. (ii) the output-inflation trade-off facing policymakers --the smaller the effect of unexpected inflation on output, the lower the inflation. When considering an open economy, the real exchange rate can also be included in the objective function, since policymakers may be concerned about export profitability. The question is how an ERM-type regime might affect the time-consistent inflation rate. In a complete information environment, if a lower time-consistent inflation rate prevails, the sacrifice ratio will be reduced. This could happen either via a change in the trade-off between inflation and output perceived by the policymakers, or via a change in the authorities' objective function. If the ERM is portrayed as a system in which the nominal exchange rate is fixed between realignments and takes the PPP value at the realignment date, then the ensuing real exchange rate appreciation between realignments makes inflation more costly compared to the case in which PPP always holds (Giavazzi and Pagano 1988) [3]. It follows that the time-consistent equilibrium inflation rate is lower than in the case in which the real exchange rate is constant. In such a model, an inflation-prone country would choose this system, although its preferences are unchanged, because an inflation reduction brings about a welfare improvement. To summarize, the increase in the costs of inflating brought about by the exchange rate arrangement constitutes the advantage of "tying one's hands", i.e., of participating in the ERM agreement.

Melitz identifies another possible source of costs of inflating, that is the measures necessary to prevent large capital outflows in the presence of inflation differentials and the subsequent realignments (Melitz 1988). Typically these consist of the maintenance of high nominal interest rates and are an increasing function of inflation. The outcome is obviously a lower time-consistent equilibrium inflation rate.

The game theoretic approach is also extended to consider the equilibrium inflation rates in different ERM members, taking into account the strategic interactions between countries (Canzoneri and Henderson 1988; Giavazzi and Giovannini 1989; *et al.*). This framework captures the purported asymmetry of the system, where Germany is the centre country [4]. The asymmetry comes from the fact that the centre country, which is the one with low inflation, chooses its monetary policy first and the inflation-prone country passively sets the same level of the money stock, in order to

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[3] In an open economy model in which PPP always holds, that is where the real exchange rate is exogenous and the authorities do not try to affect it, the resulting equilibrium inflation rate is the same as in a closed economy model.

[4] For expositional convenience, Italy is used to denote a high-inflation country while Germany is the low-inflation country.

maintain the parity. The analysis carried out by Canzoneri and Henderson (1988) (see also Giavazzi 1988) uses a two-country model, where wages are set to minimize the deviations of employment from its natural rate. The policymakers' objectives are low inflation and a desired level of employment, which is higher than the natural rate. The employment objective of the high-inflation country is, however, more ambitious (this is the source of the larger inflationary bias). It is worth emphasizing that the comparison carried out by the authors is between the equilibrium inflation rate prevailing in a free floating regime, in which the two countries engage in a competitive deflationary policy, and the fixed exchange rate system which represents the ERM. The outcome of the optimization, in the former case, is a Nash equilibrium with a deflationary bias which partly offsets the inflationary bias due to the interaction between the authorities and the private sector [5]. The optimization for the ERM-type regime results in an equal inflation rate in the two countries, which equals the closed economy outcome in the centre country. It is now possible to compare the two outcomes, i.e the free floating Nash equilibrium to the fixed exchange rate Stackelberg equilibrium. Germany's inflation rate is higher than in the benchmark case. On the other hand, the high-inflation country sees a reduction in its inflation rate, provided the discrepancy between the desired levels of employment, and hence between the inflationary biases, is relatively big compared to the incentives to affect the real exchange rate in the floating regime. The latter are in turn the greater the larger the share of imported goods and the smaller the effect of the real exchange rate on income. Furthermore, the implicit assumption which has to hold is that the centre

[5] The time-consistent inflation rate in a closed economy is therefore higher than in an open economy with free floating.

country will retain her anti-inflationary stance.

Besides modifying the output-inflation trade-off, another consequence of a fixed exchange rate regime could be a change in the policymakers' preferences. One way in which an exchange rate arrangement could bring about a change in the authorities' preferences is by introducing some form of cooperation. If this involves the maximization of an objective function which is a weighted average of the participating countries' objective functions, then the resulting inflation rate is higher than the one prevailing under German leadership with its focus on low inflation. The same reasoning would apply if countries differed with respect to desired output or unemployment levels. This is a scenario that will be more relevant for European Monetary Union (EMU), when a federal institution will conduct monetary policy (Currie *et al.* 1990).

In the context of a two-country model Collins (1988) compares alternative ways of portraying the ERM. Here only cooperative regimes are analysed, as these entail a change in the authorities' objective function. Notice that in order to isolate the effects of this factor, expectations are modelled as static. Moreover, the model does not allow for an inflationary bias associated with attempts to obtain an income level higher than the natural one. It follows that the benchmark case consists of the Nash equilibrium with only a deflationary bias, which prevails in the game between the two countries. The most important implication of cooperative behaviour is that the two countries no longer try to export inflation. When there is a cooperative fixed exchange rate regime, the government's policy is more deflationary than in the case of German leadership, i.e. in a framework equivalent to that studied by Canzoneri and

Henderson (1988) [6]. (Note that some of the above results are extremely sensitive to model specification.)

A well-known criticism of the ERM as a disinflationary device is the Walters' critique. According to Walters (1986), an exchange rate regime of the ERM type, with perfect capital mobility, entails the equalization of nominal interest rates. In the presence of inflation differentials, real interest rates would be lower (possibly negative) in high-inflation countries. The ERM would therefore have perverse effects on price performance in those countries.

In a study in reply to Walters, Miller and Sutherland (1990) describe the Walters' critique as a "case of inconsistent expectations". The analysis is intended to show that Walters' line of reasoning is flawed. With the help of a model with forward-looking wage-setters as well as financial markets, Miller and Sutherland conclude that the events described by Walters would occur only if the exchange rate system had the confidence of financial markets but not of wage-setters. In fact, if the peg is not fully credible, then a nominal interest rate differential will remain together with high inflation expectations. On the other hand, if the exchange rate arrangement enjoys full credibility, then inflation expectations will fall, leaving the real interest rate unchanged. The Walters' critique is also rejected by Giavazzi and Spaventa (1990) who argue that financial liberalization and enhanced credibility can help reduce inflation and strengthen the system. Specifically, they consider a high-inflation country which removes capital controls. The initially higher nominal interest rates attract capital inflows that, on one hand, lead to the appreciation of the currency, but

[6] The explanation being that Germany now takes into account the higher initial inflation rate prevailing in Italy.

on the other, tend to eliminate the nominal interest rate differential, so that there is a lowering of real interest rates. Moreover, another effect of liberalization is that more agents have access to foreign capital markets where the cost of borrowing is lower. In this scenario, initially, inflation-prone countries do experience a lower real interest rate. However, Giavazzi and Spaventa claim that the subsequent initial burst in prices makes the loss in competitiveness even larger, and provided this effect is stronger relative to the effect of the real interest rate on aggregate demand, inflation will decrease more quickly.

The literature surveyed here cannot provide a clear-cut answer to the question of whether a pegged exchange rate system can either improve inflation performance or permit a disinflation at a lower cost. The results of the theoretical studies depend on the framework chosen to portray the fixed exchange rate arrangement, the model used, and the objective function.

The debate about the effectiveness of a pegged exchange rate as an anti-inflationary device is particularly lively because some countries participating in the ERM experienced a very slow decline in inflation. Some argue that the public's slow adjustment of expectations, the learning process, could explain the lengthy disinflation observed in some European countries after joining the ERM. Driffill and Miller (1992) and Giovannini (1990) deal with learning following an exchange rate regime shift similar to that assumed here.

In Driffill and Miller's paper, the public learns about the probability of the occurrence of a realignment. The private sector starts from a high prior, and then gathers information from the observation of realignments. The Bayesian learning process is then embodied in a model of overlapping contracts *a' la* Calvo (1983). The contracts reflect the likelihood of a realignment and directly affect actual inflation. Due to the initial high probability attached to the occurrence of a realignment, and to the gradual revision of expectations, actual inflation declines slowly, competitiveness worsens and output descends below full capacity.

Giovannini (1990) adopts a Barro-Gordon type model, where inflation is replaced by the exchange rate and the latter becomes the policymakers' choice variable. Once again it is assumed that a change in the exchange rate arrangement takes place, from floating, in which the government manipulates the exchange rate, to a fixed rate. Expectations about the exchange rate are formed looking at past experience and result in being initially higher than the actual value, but they adjust gradually, provided the parity is not changed. As a result, an output loss is incurred. A criticism which can be made of both studies is that they neglect the interaction of the public's expectations with the government's strategy, i.e. policy is not endogenous.

Others point out that ERM membership performs a signalling role of the new anti-inflationary stance and that a credibility bonus would ensue for inflation-prone countries. Along similar lines, some highlight the possibility that governments implement policies which convey information about their preferences to private agents, namely they signal their priorities amongst the different objectives. Britton (1992) discusses the importance of connecting the private sector's revision of beliefs with the policymakers' strategy in order to understand the consequences of a policy shift, like the one occurring when a country joins the ERM.

No analytical model applied to this context has been developed embodying both rational learning and endogenous policy. Britton suggests that a rigorous theory of forecasting and endogenous policy of realignments would be very useful, but at the same time, he points out that it is unlikely that this can be done, due to the technical difficulties involved. This chapter attempts to close this gap existing in the literature. The aim is to see whether the thesis that learning can produce a lengthy disinflation is defensible on theoretical grounds, in the context of the model developed below. In particular, the purpose of this chapter is to analyse how the change in the exchange rate regime and the asymmetric information structure affect the public's inflationary expectations and the authorities' decision to create inflation.

It is assumed that, in the initial exchange rate system, PPP always holds. The public cannot gather information about the policymakers' preferences about competitiveness, due to the fact that the crawling of the nominal exchange rate offsets domestic inflation, keeping the real exchange rate constant. Thus, the policymakers' decision to create inflation cannot affect this variable. However, once the new regime is in place, the government's choice to inflate entails worsened competitiveness and thus reflects its intentions with respect to the real exchange rate. The public can now learn about the authorities' willingness to endure misalignments in the competitive position of their country. At the same time, since the government is aware of the private sector's learning, it will take into account the information content of its actions in deciding the optimal policy. The approach and the techniques adopted here are the same as the ones used in Chapter 2.

The basic model is an open economy version of the Barro-Gordon model (Barro and Gordon 1983a) as used by Giavazzi and Pagano (1988), into which stochastic realignments are introduced. This appears to be the natural framework in which to imbed the asymmetric information feature and to analyse the interactions between the authorities and the public. The model consists of a dynamic non-cooperative game of asymmetric information between two players, the government and the private sector. This setup can also be used to examine what would happen when shifting to a monetary union.

This chapter is organized as follows. In Section 3.2, the basic model, which includes stochastic realignments, is specified and the solution for the symmetric information case is derived. Section 3.3 introduces the asymmetric information structure and illustrates the solution procedure. In Section 3.4, the public's learning problem is described. In the Section 3.5, the government's optimal policy is worked out. Section 3.6 concludes.

3.2. The basic model

The basic model is an amended version of the Giavazzi and Pagano model with full capital mobility (Giavazzi and Pagano 1988). The novelty is the introduction of stochastic realignments and the government's imperfect control of inflation via the fiscal lever. The time-consistent inflation rate which results in the ERM type regime is lower than that prevailing under floating with PPP.

The model includes two goods: one is traded and its price, p^*, is that prevailing on international markets and is an exogenous variable. The price of the tradable good in domestic currency depends also on the exchange rate at time t, s_t, defined as the price of foreign currency in terms of the domestic currency. The other good is not traded and its price, p_t, is set as a mark-up over wages, w_t; for simplicity p_t equals w_t. Wages are homogenous in the two sectors of the economy, the tradable and the non-tradable, and wage setting is based on the price level expected by private agents for time t, i.e. p^e_t, on the level of the demand, y_t, and on a supply shock, z_t, modelled as a Weiner process, which cannot be immediately offset by the government. Aggregate demand depends on real government spending, $g_t - p_t$. Competitiveness, or the real exchange rate, q_t, is an increasing function of p^* and s_t and a decreasing function of p_t. Variations in competitiveness lead to changes in the composition of output. The profitability of the tradable sector, in turn, is directly related to competitiveness.

As it will be seen later, competitiveness will enter the government's objective function[7]. All this can formalized in logarithms as follows:

$$p_t = w_t \tag{1}$$

$$w_t = p_t^* + \beta y_t + z_t \tag{2}$$

$$y_t = g_t - p_t \tag{3}$$

$$q_t = p^* + s_t - p_t \tag{4}$$

The core of the model is the non-cooperative game between two players, the government and the private sector. The duration of the game is infinite; it is a dynamic game. In this section, symmetric information is assumed and the only

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[7] This treatment of competitiveness is the same as that of Giavazzi and Pagano. It would also be possible to make aggregate demand dependent on competitiveness, i.e. introduce a q_t term in equation (3). This would then modify equation (6) and (7) respectively as follows:

$$p_t = \frac{1}{\left[1+\beta(1+\delta)\right]}\left[\beta g_t + p_t^* + z_t + \beta\delta(p^* + s_t)\right]$$

$$dp_t = \frac{1}{\left[1+\beta(1+\delta)\right]}\left(\beta dg_t + dp_t^* + dz_t\right).$$

Although nothing would change in the rest of the model, in this version of the model, output would decline between realignments, if inflation is positive. It would then go back to full employment when a realignment occurs, as this restores competitiveness. It follows that the level of economic activity would have a saw-tooth path. On the other hand, in the version adopted in the main body of the chapter, output is, on average, at its full employment level.

uncertainty is represented by the supply shock. Later, the effects of the public's ignorance are examined. Both players choose their actions by optimizing their objective function over an infinite horizon. The private sector's objective is to minimize the mean square error of the inflation forecast [8]. The public moves first by fixing its strategy, which is the expectation of the price level, p^e_t, or equivalently the expectation of inflation, π^e_t. Then, the government fixes its strategy, which is nominal public spending, g_t. Before specifying the objective function of the government, it is useful to reduce the part of the model spelled out so far to two equations. Substituting equation (1) into equation (2) and rearranging, I obtain the first reduced form equation, i.e.:

$$y_t = \frac{1}{\beta}\left(p_t - p_t^e\right) - \frac{1}{\beta}z_t = \frac{1}{\beta}\left(\pi_t - \pi_t^e\right) - \frac{1}{\beta}z_t\,, \tag{5}$$

which shows that output is a function of price, or inflationary, surprises, and it increases with both of these. Furthermore, output depends negatively on the supply shock. Equation (5) can now be equated to the right hand side of the aggregate demand equation, equation (3). This yields the second equation of the reduced form, that is:

$$p_t = \frac{1}{(1+\beta)}\left[\beta g_t + p_t^e + z_t\right]. \tag{6}$$

The above equation shows that the price level is affected by the actions of both

[8] The public's objective function is specified in the next section in the sub-section on learning.

players, namely by the public's price expectations and the government's choice of public spending. Notice also that the supply shock, z_t, affects the price level, so that, given the public's strategy, the authorities' choice of g_t does not determine a unique value for p_t, but there is a whole distribution of values. Equation (6) can be rewritten in terms of rates of change and it results in being:

$$dp_t = \frac{1}{(1+\beta)}\left(\beta dg_t + dp_t^e + dz_t\right). \tag{7}$$

In the rest of the paper instead of referring to g_t as the strategy of the government, for simplicity, the choice of planned inflation, π^p_t, is the authorities' strategy [9]. The distinction between planned inflation, which is determined by the government, and actual inflation is necessary because of the government's imperfect command of inflation, carried out by fiscal policy. Actual and planned inflation differ by a white noise disturbance, i.e. the increment of a Weiner process, so that the evolution of the price level is described by the following equation:

$$dp_t = \pi^p_t\, dt + dz_t\,. \tag{8}$$

In Giavazzi and Pagano's paper, inflation is controlled by monetary growth. However, assuming perfect capital mobility and a fixed exchange rate [10], the choice of inflation can no longer be made via monetary policy, which becomes endogenous

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[9] The strategy space, which comprises all admissible strategies, is the same for the two players and is the real line.

[10] This proposition is true even if the exchange rate system is not exactly fixed, but allows infrequent adjustments of the parity, like the ERM, although the degree of the endogeneity of monetary policy would be reduced.

(see also Obstfeld 1988; Winckler 1991). For this reason, I make inflation a function of the fiscal lever and this calls for imperfect controllability of inflation.

The government's objective function can now be put forward. The authorities like to create inflation surprises, as these raise output. They also dislike inflation *per se* and its damaging effects on the profits of the export sector, which depend on the level of competitiveness. The government maximizes the following objective function:

$$V_0 = E_{gov}\left[\int_0^\infty e^{-\rho t}\left[hq_t + c(\pi_t - \pi_t^e) - \frac{1}{2}(\pi_t^p)^2\right]dt\right], \tag{9}$$

where ρ is the authorities' discount rate, and E_{gov} is the expectation operator with respect to the government's information set (which, in this basic model, coincides with the public's information set). The control variable is π^p_t and the state variable of this problem is q_t, i.e. the real exchange rate.

The real exchange rate can also be expressed in the following way:

$$q_t = q_0 - \int_{t_l}^{t}\pi_s ds , \tag{10}$$

where q_0 is the level of competitiveness at time 0, i.e. when cumulated inflation is 0, which equals p^* plus s_0, and is assumed to be the level of the real exchange rate prevailing in the floating regime, that is the PPP level. As time goes by, competitiveness is eroded by previous inflation.

Realignments are the other crucial variable, besides domestic inflation, in determining the real exchange rate. In equation (10), t_l denotes the date of the last realignment, so that the integral represents cumulated inflation from that date. This is so because it is assumed that at each realignment date the initial level of competitiveness, q_0 , is restored. The modelling of realignments put forward here differs from Giavazzi and Pagano's. In their paper, realignments occur at given dates, with a fixed time interval between them, even when full capital mobility is allowed for [11]. However, it is not clear how to reconcile perfect capital mobility with fully anticipated realignments. Here, this problem is overcome by introducing stochastic realignments. A Poisson process has been chosen to model realignments because it captures the fact that realignments occur infrequently. The random variable X, denoting the number of realignments, is distributed as a Poisson process. The probability of having a parity change is constant over time and, in an interval of time which tends to 0, equals [12]

$$\text{prob}(X=1) = \lambda\Delta t + o(\Delta t) \tag{11}$$

where λ is the parameter of the Poisson process, which is an institutional feature of the monetary system, and hence, outside the authorities' discretion.

The fact that at each realignment the level of competitiveness q_0 is re-established, combined with stochastic parity changes and imperfect controllability of inflation, entails that the size of the realignment is also a random variable.

Initially, each period between realignments is treated independently from any

[11] In Giavazzi and Pagano's paper, when full capital mobility is introduced, the government's objective function is modified to capture the dislike for interest rate variability induced by parity changes.

[12] The probability of more than one event occurring in an interval of time which tends to zero, is zero.

other period. This is intended to model a situation in which, after each realignment, everything goes back to the beginning. Later in this section, it will be shown that, in the deterministic case, the solution obtained when each period between realignments is treated independetly holds also when links between realingments are taken into account.

When everything goes back to the beginning after each realignment, the problem becomes separable between parity changes. Therefore, only one of these intervals of time is considered, as the problem is the same in each of them. The government's objective function is modified accordingly, that is neglecting temporarily the distinction between π_t and π^e_t by assuming that π_t is a deterministic process, it results in being;

$$V_0 = E_{gov} \left[\int_0^{\tau_N} e^{-\rho t} \left[hq_t + c(\pi_t - \pi^e_t) - \frac{1}{2}(\pi_t)^2 \right] dt \right] , \tag{12}$$

where the date of the last realignment has been normalized to 0 and τ_N denotes the date of the next realignment. The latter is a stopping time defined by:

$$\tau_N = \inf \{ t > 0;\ X_t = 1 \} , \tag{13}$$

in words, it is the first time the random variable X takes the value 1. Before proceeding with the optimization, it is necessary to calculate the expectation of the whole objective function, which is a function of the random time. The result is presented in the lemma below.

Lemma 1

Given that the density of the random time τ_N is:

$$\lambda \, e^{-\lambda \tau_N} , \tag{14}$$

and letting

$$Q_t = hq_t + c(\pi_t - \pi_t^*) - \frac{1}{2}\pi_t^2 , \tag{15}$$

the expectation of the objective function with respect to the next realignment date is:

$$\int_0^\infty e^{-(\rho+\lambda)t} \, Q_t \, dt \ . \tag{16}$$

Proof

The expectation of the objective function with respect to the random time can be computed as follows:

$$E_{\tau_N} \left[\int_0^{\tau_N} e^{-\rho t} \, Q_t \, dt \right] = \int_0^\infty \lambda e^{-\lambda \tau_N} \left[\int_0^{\tau_N} e^{-\rho t} \, Q_t \, dt \right] d\tau_N \ . \tag{17}$$

Equation (16) is obtained by reversing the order of integration in the right hand side of equation (17).

The problem has thus been transformed from a stochastic one, due to the presence of a stopping time as the horizon of the optimization, into a deterministic one, where the parameter of the Poisson process, λ, appears as additional discounting in the government's objective function [13].

The government's optimization can now be tackled. Formally, the problem is the following:

$$\underset{\pi_t}{\text{Max}} \int_0^{\infty} e^{-(\rho+\lambda)t} \left[hq_t + c(\pi_t - \pi_t^e) - \frac{1}{2}\pi_t^2 \right] dt \tag{18}$$

$$\text{s.t.} \qquad \dot{q} = -\pi_t \,, \tag{19}$$

where equation (19) is obtained by differentiating equation (10) with respect to time. As already mentioned, π_t is the control variable and q_t is the state variable. The solution which satisfies the equilibrium condition $\pi_t = \pi^e_t$, namely the time-consistent solution for the inflation rate, denoted by π^*_t, can be easily obtained as:

$$\pi_t^* = c - \frac{h}{(\lambda+\rho)} \,. \tag{20}$$

The optimal inflation rate is time-invariant and shows that a country has always lower inflation when participating in an exchange rate arrangement like the ERM than under a free floating regime with PPP. In fact, under the latter regime the real exchange rate cannot be affected by policymakers' actions, so that the time-consistent inflation rate coincides with the closed economy solution, which is $\pi^*_t = c$. In the solution obtained here, under the pegged exchange rate regime, the reduction in the inflationary bias is larger, the smaller are the discount rate and the probability of

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[13] If the simplifying assumptions of having $\pi_t = \pi^e_t$ and π_t deterministic were removed, the problem would still be stochastic. In the rest of this paragraph these assumptions are maintained because the solution for the optimal policy is the same, whilst the distinction between actual and planned inflation and the stochastic nature of π_t will be reintroduced in the next paragraph.

having a realignment [14]. This makes sense because if realignments are more likely, and hence the expected date of the next realignment is closer, the period during which inflation costs are incurred is shorter. Hence, the tighter the institutional constraint is, the greater is the benefit of lower inflation resulting from "tying one's hands". Also, the larger h is, i.e. the more the government cares about competitiveness, the lower inflation is.

In order to prove that the same solution results in the case in which the links between realignment period are taken into account, it is necessary to consider the government's objective function composed of the infinite sequence of periods between parity changes, i.e.:

$$V_0 = E_{gov} \left[\sum_{i=0}^{\infty} \int_{\tau_i}^{\tau_{i+1}} e^{-\rho t} Q_t \, dt \right] , \qquad (21)$$

where τ_i denotes the time of the ith realignment. τ_i is a stopping time defined by:

$$\tau_i = \inf \ [t>0;\ X_t = i] \ ; \qquad \tau_0 = 0 \ . \qquad (22)$$

The above equation means that τ_i is the first time that the variable X (which is the number of realignments) takes the value i. The calculation of the expectation of the government's objective function, which is a function of the random times of parity changes, is presented in the lemma below, maintaining the assumption that π_t is deterministic.

[14] Notice that in this model it is not necessarily the case that there is a positive inflationary bias; in fact it is possible to have 0 as a time consistent inflation rate, or even a negative rate.

Lemma 2

Given that the density of the random time τ_i is:

$$\tau_i \sim \frac{\lambda\ (\tau_i\lambda)^{i-1}\ e^{-\lambda\tau_i}}{(i-1)!}, \tag{23}$$

the expectation of the government's objective function with respect to the realignments dates is:

$$\int_0^{\infty} e^{-(\rho+\lambda)t}\ Q_t\ dt + \int_0^{\infty} e^{-\rho s}(1-e^{-\lambda s})\ \bar{Q}_s\ ds; \qquad s>\tau_1. \tag{24}$$

Where

$$\bar{Q}_s = hq_s + c(\pi_s-\pi_s^e) - \frac{1}{2}\pi_s^2; \qquad s>\tau_1. \tag{25}$$

Proof

Firstly, equation (21) is rewritten as[15]:

$$E_{\tau_i}\left[\int_{t=0}^{\tau_1} e^{-\rho t}\ Q_t\ dt\right] + E_{\tau_i}\left[\sum_{i=1}^{\infty}\ \int_{\tau_i}^{\tau_{i+1}} e^{-\rho s}\ \bar{Q}_s\ ds\right]. \tag{26}$$

The expectation of the first element in equation (26) has already been computed in lemma 1. Hence, the expectation of the first element appearing in the summation in equation (26) is calculated in the following way:

[15] Notice that in what follows s always denotes a time after the first realignment.

$$E_{gov}\left[\int_{\tau_1}^{\tau_2} e^{-rs}\bar{Q}_s ds\right] = E\left[\int_0^{\tau_2} \lambda e^{-\lambda\tau_1}\left[\int_{\tau_1}^{\tau_2} e^{-rs}\bar{Q}_s ds\right] d\tau_1\right], \tag{27}$$

where the right hand side term involves the product of the element whose expectation has to be calculated times the density of the time of the first realignment. By reversing the order of integration, this computation results in:

$$E\left[\int_0^{\tau_2} e^{-rs}\,(1-e^{-\lambda s})\,\bar{Q}_s\,ds\right]. \tag{28}$$

The expectation shown in equation (28) is then computed as follows:

$$E\left[\int_0^{\tau_2} e^{-rs}\,(1-e^{-\lambda s})\,\bar{Q}_s\,ds\right] = \int_0^{\infty}\frac{\lambda\,(\tau_2\lambda)\,e^{-\lambda\tau_2}}{1!}\left[\int_0^{\tau_2} e^{-rs}\,(1-e^{-\lambda s})\,\bar{Q}_s\,ds\right] d\tau_2\,. \tag{29}$$

In order to solve the double integral in equation (29), the order of integration is reversed and the integral in τ_2 is computed using the general formula shown below.

$$\int_s^{\infty}\frac{\lambda\,(\tau_i\lambda)^{i-1}\,e^{-\lambda\tau_i}}{(i-1)!}\,d\tau_i = \sum_{j=0}^{i-1}\frac{(\lambda s)^j\,e^{-\lambda s}}{j!}\,. \tag{30}$$

It follows that

$$E_{gov}\left[\int_{\tau_1}^{\tau_2} e^{-rs}\,\bar{Q}_s\,ds\right] = \int_0^{\infty} e^{-rs}\,(1-e^{-\lambda s})\,(e^{-\lambda s}+\lambda s e^{-\lambda s})\,\bar{Q}_s\,ds\,. \tag{31}$$

The expectation of subsequent elements of the summation in equation (21) can be worked out in the same fashion. V_0 can thus be written as:

$$
\begin{aligned}
V_0 = \int_0^{\infty} e^{-(\rho+\lambda)s}\, Q_s\, ds + \int_0^{\infty} e^{-\rho s} (1-e^{-\lambda s})(e^{-\lambda s}+\lambda s e^{-\lambda s})\, \bar{Q}_s\, ds\, + \\
\int_0^{\infty} e^{-\rho s} (1-e^{-\lambda s})(1-e^{-\lambda s}-\lambda s e^{-\lambda s})(e^{-\lambda s}+\lambda s e^{-\lambda s}+\frac{(\lambda s)^2}{2!} e^{-\lambda s})\, \bar{Q}_s\, ds + \ldots
\end{aligned}
\tag{32}
$$

Equation (32) is simplified by noting that if a new variable, $f_i(s)$, defined as:

$$
f_i(s) = 1 - e^{-\lambda s} - \lambda s e^{-\lambda s} - \ldots - \frac{(\lambda s)^i}{i!} e^{-\lambda s} \tag{33}
$$

is introduced, the right hand side of equation (32) can be expressed in the following form:

$$
V_0 = \int_0^{\infty} e^{-(\rho+\lambda)s}\, Q_s\, ds + \int_0^{\infty} e^{-\rho s}\, \bar{Q}_s \left[f_0(1-f_1)+f_0 f_1(1-f_2)+f_0 f_1 f_2(1-f_3)+\ldots\right] ds\ . \tag{34}
$$

Once the appropriate cancellations are made, equation (24) results and the proof is completed.

In this deterministic world, since competitiveness is restored to its initial level q_0 after each realignment, Q_s cannot be affected by government's decisions taken before the first realignment. Thus, the optimization of the objective function appearing in equation (24) is equivalent to the optimization of the objective function presented in equation 16. It follows that the government's optimal policy is the same in each period between realignments.

The separability between realignments does not hold any longer when the problem is stochastic and asymmetric information is present. In this latter case, due to the continuing learning process of the public, the government policy affects the initial conditions of the game in subsequent periods. In particular, it affects the public's estimate of the parameter h and the precision of this estimate. In the following section, where asymmetric information and learning enter the analysis, it is assumed that after each realignment everything goes back to the beginning, due for example to the fact that the incumbent government resigns and a new one comes to power. Hence, each period between realingments is treated independently from any other period. This assumption is adopted because it simplifies the solution procedure without affecting the results in any substantial way.

Finally, this model can also be used to derive the optimal time-consistent inflation rate in a monetary union, π_t^{**}, in which realignments are ruled out and hence lambda equals zero. The solution for the time consistent inflation rate is:

$$\pi_t^{**} = c - \frac{h}{\rho} . \tag{35}$$

From the above equation, it is possible to see that inflation would be lower in a monetary union than in the ERM-type regime examined above. This is explained by the fact that in the former case inflation entails a permanent loss in competitiveness and thus greater costs are incurred than in a pegged exchange rate regime. Notice that in a monetary union, since the loss of competitiveness cannot be recovered, in order to prevent the accumulation of current account deficits, it will be necessary to embody additional features in the model which constrain the inflation rate from diverging from that of other member countries of the union. An example of this might be to make the parameter, h, an increasing function of the level of the real exchange rate.

3.3 The model with asymmetric information: the information structure and the solution procedure

In this section, the information asymmetry between the government and the private sector is introduced, as already sketched out in the introduction.

In a country which has recently changed the exchange rate regime, from floating to a peg, it may be the case that the public does not know how much the government cares about competitiveness [16]. It seems reasonable to introduce the private sector's uncertainty in the parameter h of the authorities' objective function, because the public never had the opportunity to gather information about this preference parameter in the flexible exchange rate regime with PPP. It is assumed that the distribution from which h has been drawn is common knowledge and it is:

$$h \sim N(\bar{h}, \sigma_h^2), \tag{36}$$

where the mean of the distribution is the public's prior and σ_h^2 is the variance of the distribution, which is an indicator of the public's degree of uncertainty.

In this section, the assumption of imperfect controllability of inflation by the

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[16] As already pointed out in the introduction, in the floating regime assumed here, PPP holds at all times, so that the government cannot affect the real exchange rate. In the pegged exchange rate system instead, the government's decision to create inflation reduces competitiveness.

$$h \sim N(\bar{h}\ ,\ \sigma_h^2)\ , \tag{36}$$

where the mean of the distribution is the public's prior and $\sigma^2{}_h$ is the variance of the distribution, which is an indicator of the public's degree of uncertainty.

In this section, the assumption of imperfect controllability of inflation by the government is made; thus the evolution of the price level, or actual inflation, is as shown in equation (8). Furthermore, planned inflation, which is the authorities' strategy, is the government's private information, whereas actual inflation is observed by all agents in the economy. It is clear that in this set up the information sets of the two players differ: the public's information set is a subset of the government's one and compared to the latter it lacks h and $\pi^p{}_t$.

From the observation of actual inflation, the private sector gathers information, using an optimal Kalman-Bucy filter procedure, about h, the unknown element of the policymakers' preferences, and hence optimally revises its strategy, namely the expected inflation rate.

The analysis is aimed at examining how the public's inflationary expectations and the government's choice of optimal planned inflation are modified by the asymmetric information structure and the stochastic environment, given that both players act strategically.

The public's filtering problem and the government's optimization can be solved in a sequence of stages because the separation principle applies; it applies given the linearity of the stochastic differential equations, which represent the constraints, and the fact that the objective function is quadratic in the control and linear in the state variables (see Davis 1977). The solution procedure is the same as the one used in the previous chapter. It begins by postulating the government's optimal policy. The following form is postulated:

$$\pi_t^p = cA_t - \frac{h}{(\lambda+\rho)}B_t \ , \tag{37}$$

where A_t and B_t are undetermined coefficients which will be derived once the government's optimal strategy has been solved for. A_t and B_t are assumed to be common knowledge.

The stochastic differential equation which describes the public's updating of beliefs, which is the evolution of $\hat{h}_t$, becomes part of the policymakers' optimization. In fact $\hat{h}_t$ becomes the second state variable, in addition to q_t , in this asymmetric information context. The resulting equilibrium is a rational expectations equilibrium and the equilibrium of this game is a perfect bayesian equilibrium.

3.4 The public's learning

The public uses a Bayesian algorithm, i.e. the Kalman-Bucy filter, to obtain and revise the estimate of the parameter h and then sets its strategy as:

$$\pi_t^e = cA_t - \frac{\hat{h}_t}{(\lambda+\rho)}B_t \ . \tag{38}$$

The chosen strategy is measurable with respect to the information field generated by all the variables of the model which can be observed by private agents. It minimizes the public's objective function, which is:

$$J = E\left[\int_0^\infty (\pi_t - \pi_t^e)^2 \, dt\right] . \quad (39)$$

In other words, the private sector aims at minimizing the mean square error of the forecast of inflation, which, given the information structure assumed here, is equivalent to minimizing the mean square error of the forecast of the parameter h. Formally, the filtering can be expressed by two equations. The first, equation (40), describes the system, which is the parameter h. The second equation (41) describes the evolution of the observations, namely actual inflation or the variations of the price level. The equations are as follows:

$$dh = 0 ; \qquad E_0 (h) = \bar{h} , \ E_0 \left[h - E_0 (h)\right]^2 = \sigma_h^2 . \quad (40)$$

$$dp_t = \pi_t^p dt + dz_t = \left[cA_t - \frac{h}{(\lambda+\rho)} B_t\right] dt + dz_t . \quad (41)$$

Since the above equations are linear, the estimate can be computed with the Kalman-Bucy procedure. The estimate of h, $\hat{h}_t$, is the best estimate based on the available observations. In fact, the filter at time t is adapted to the σ-algebra generated by the continuous observations of actual inflation from time 0 to time t and minimizes the mean square error of the forecast of h. The filter, $\hat{h}_t$, satisfies the following stochastic differential equation:

$$d\hat{h}_t = - \frac{B_t S_t}{(\lambda+\rho)} \left[dp_t - \left[cA_t - \frac{B_t}{(\lambda+\rho)} \hat{h}_t\right] dt\right] . \quad (42)$$

where $S_t = E[(h - \hat{h}_t)^2]$ solves the deterministic Riccati equation shown below:

$$\dot{S}_t = -\frac{B_t^2 S_t^2}{(\lambda+\rho)^2} ; \qquad S_0 = E[h-E_0(h)]^2 = \sigma_h^2 . \tag{43}$$

The solution to equation (43) is:

$$S_t = \left[\frac{1}{(\lambda+\rho)^2}\int_0^t B_s^2 ds + \frac{1}{\sigma_h^2}\right]^{-1} = K_t^{-1} . \tag{44}$$

Notice that the speed at which the public's learning takes place depends on two things, besides the parameters of the model. i) the term in brackets in equation (42), which is the discrepancy between actual and expected inflation, and hence, between the true and the estimated value of h. This element becomes smaller and smaller as time goes by, because the public is learning. The learning process is thus faster at the beginning when each observation conveys a lot of information, and slows down over time. ii) B_t , that is the undetermined coefficient to be chosen by the government. The smaller B_t is, the slower the adjustment of expectations, because the greater becomes the relative size of the noise in the observations [16]. Likewise, the larger B_t is, the stronger the signal the government sends, and hence, the easier for the public to extract information. Finally, solving the stochastic differential equation of the filter, the following expression for the estimate of h is obtained:

[16] The B_ts affect also S_t in an inverse way; the larger the B_ts are, the smaller S_t is; however, if one considers only one B_t at a time, keeping the others fixed, the effect on S_t is infinitesimal, whereas there is obviously a finite effect on B_t and therefore the latter effect dominates.

$$\hat{h}_t = \frac{\frac{1}{(\lambda+\rho)}\left[c\int_0^t B_s A_s ds - \int_0^t B_s dp_s\right] + \frac{\bar{h}}{\sigma_h^2}}{\frac{1}{(\lambda+\rho)^2}\int_0^t B_s^2 ds + \frac{1}{\sigma_h^2}} . \tag{45}$$

It is possible to see that at time 0, the public's estimate of h is just the prior; as time tends to infinity, i.e. in the steady state, $\hat{h}_t$ tends to the true value.

Notice that the public sector's strategy, π^e_t, is not fully worked out yet, since it includes both A_t and B_t which are unknown so far, and will only be determined in the government's optimization. This is done in the next section.

3.5. The government's optimization

The government's optimization problem in the asymmetric information environment can now be tackled, as all the elements needed are available. These are: i) the government's objective function; ii) the evolution of the first state variable, competitiveness; iii) the evolution of the second state variable, the public's estimate of the parameter h. The second state variable is newly introduced in the asymmetric information case, because the authorities now have to take into account the effects of their decisions on the public's learning process. The strategic interactions between the two players can thus be examined.

Formally, the government's optimization is:

$$\max_{\pi_t^p} \; E_{gov}\left[\int_0^\infty e^{-(\lambda+\rho)t}\,[hq_t + c(\pi_t - \pi_t^e) - \frac{1}{2}(\pi_t^p)^2\,]\,dt\right] , \tag{46}$$

$$\text{s.t.} \qquad dq_t = -\pi_t^p\,dt - dz_t\,, \tag{47}$$

$$d\hat{h}_t = -\frac{B_t}{(\lambda+\rho)K_t}\left[\left[\pi_t^p - \left[cA_t - \frac{B_t\,\hat{h}_t}{(\lambda+\rho)}\right]\right]dt + dz_t\right]\,, \tag{48}$$

where π^e_t is given by equation (38) and K_t is defined in equation (44). The solution procedure involves the use of the HJB equation. It provides the optimal government policy, that is the optimal path for planned inflation, given the private sector's strategy. Since the government's decision plan satisfies the HJB, it is time-consistent. The strategy is measurable with respect to the policymakers' information field, generated by the history of all the variables of the model, excluding the current value of the supply shock. The resulting equilibrium is a perfect bayesian equilibrium.

Proposition

The equilibrium strategy of the government, $(\pi^p_t)^*$, obtained from the game outlined above is:

$$(\pi_t^p)^* = c\left[1 - \frac{e^{(\lambda+\rho)t}}{(\lambda+\rho)^2}\left[\bar{C} - \int_0^t \frac{e^{-(\lambda+\rho)s}}{K_s}\,ds\right]\right] - \frac{h}{(\lambda+\rho)} \tag{49}$$

$$\text{where} \quad \bar{C} = \lim_{t\to\infty} \int_0^t \frac{e^{-(\lambda+\rho)s}}{K_s}\,ds\,. \tag{50}$$

Proof

The solution is derived by postulating the following form for the value function:

$$V_t\,(q_t\,,\,\hat{h}_t) = e^{-(\lambda+\rho)t}\,W_t\,(q_t,\,\hat{h}_t)\,, \tag{51}$$

where W_t is a linear function of the state variables, q_t and $\hat{h}_t$, that is:

$$W_t = \mu_{0t} + \mu_{1t}\, q_t + \mu_{2t}\, \hat{h}_t\ . \tag{52}$$

The HJB equation is then a partial differential equation in V and results in being:

$$\begin{aligned}(\lambda+\rho)W_t - \dot{\mu}_{0t} - \dot{\mu}_{1t}\, q_t - \dot{\mu}_{2t}\, \hat{h}_t = \\ \max_{\pi_t^P}\ c\pi_t^P - c\left[cA_t - \frac{\hat{h}_t B_t}{(\lambda+\rho)}\right] - \frac{1}{2}(\pi_t^P)^2 + hq_t - \mu_{1t}\pi_t^P \\ - \mu_{2t}\,\frac{B_t}{(\lambda+\rho)K_t}\left[\pi_t^P - \left[cA_t - \frac{\hat{h}_t B_t}{(\lambda+\rho)}\right]\right]\ ;\end{aligned} \tag{53}$$

from which the first order condition yields:

$$(\pi_t^P)^* = c - \mu_{1t} - \frac{\mu_{2t} B_t}{(\lambda+\rho)K_t}\ . \tag{54}$$

The next step consists of the determination of the undetermined coefficients of the value function, μ_{1t} and μ_{2t}, by equating the terms in q_t and in $\hat{h}_t$ respectively, on the left hand side and the right hand side of the HJB equation. This gives two deterministic differential equations, one in μ_{1t} and the other in μ_{2t}. The solutions to these equations, which are fully worked out in Appendix B, are respectively:

$$\mu_{1t} = \frac{h}{(\lambda+\rho)} \,, \tag{55}$$

$$\mu_{2t} = \frac{e^{(\lambda+\rho)t}\, K_t c}{(\lambda+\rho)} \left[\bar{C} - \int_0^t \frac{B_s e^{-(\lambda+\rho)s}}{K_s} \, ds \right] \tag{56}$$

where

$$\bar{C} = \lim_{t\to\infty} \int_0^t \frac{B_s e^{-(\lambda+\rho)s}}{K_s} \, ds \,. \tag{57}$$

The coefficients μ_{1t} and μ_{2t} are then substituted in the optimal control, equation (54). The government's strategy results in being of the form initially postulated in equation (37), so that it is the rational expectations equilibrium solution, which will be complete after the determination of the coefficients A_t and B_t, which appear in the conjectural function. The solutions for A_t and B_t are shown below.

$$B_t = 1 \,; \tag{58}$$

$$\begin{aligned} A_t &= 1 - \frac{e^{(\lambda+\rho)t}\, B_t}{(\lambda+\rho)^2} \left[\bar{C} - \int_0^t \frac{e^{-(\lambda+\rho)s} B_s}{K_s} \, ds \right] = \\ &1 - \frac{e^{(\lambda+\rho)t}}{(\lambda+\rho)^2} \left[\bar{C} - \int_0^t \frac{e^{-(\lambda+\rho)s}}{K_s} \, ds \right] < 1 \,. \end{aligned} \tag{59}$$

The two equations above, substituted in equation (40) to give equation (57), complete the proof.

A_t and B_t can now be substituted also in the equations of the learning process.

For convenience, I rewrite the equilibrium strategies of the two players:

i) the government's strategy:

$$(\pi_t^p)^* = c\left[1 - \frac{e^{(\lambda+\rho)t}}{(\lambda+\rho)^2}\left[\bar{C} - \int_0^t \frac{e^{-(\lambda+\rho)s}}{K_s}\,ds\right]\right] - \frac{h}{(\lambda+\rho)}\;; \tag{60}$$

ii) the private sector's strategy:

$$(\pi_t^e)^* = c\left[1 - \frac{e^{(\lambda+\rho)t}}{(\lambda+\rho)^2}\left[\bar{C} - \int_0^t \frac{e^{-(\lambda+\rho)s}}{K_s}\,ds\right]\right] - \frac{\hat{h}_t}{(\lambda+\rho)}\;, \tag{61}$$

where

$$\hat{h}_t = \frac{\dfrac{ht}{(\lambda+\rho)^2} + \displaystyle\int_0^t dz_s + \dfrac{\bar{h}}{\sigma_h^2}}{K_t}\;, \tag{62}$$

$$K_t = \frac{t}{(\lambda+\rho)^2} + \frac{1}{\sigma_h^2}\;, \tag{63}$$

and

$$\bar{C} = \lim_{t\to\infty} \int_0^t \frac{e^{-(\lambda+\rho)s}}{K_s}\,ds\;. \tag{64}$$

Some observations on the optimal government's policy are now warranted.

First of all, the coefficient, A_t, is less than one; this means that planned inflation is lower in the presence of asymmetric information than in the case of symmetric information. The informational advantage offers the possibility to the government to sustain lower time-consistent inflation rates.

The second observation concerns B_t which is unity; this shows that the second term in the planned inflation formula (equation 60) is unchanged with respect to the symmetric information scenario (see equation 20). This second element of the government's choice of inflation appears with the introduction of the pegged exchange rate regime and can be labelled the "discipline effect" of the ERM type system. The finding that B_t equals one signifies that the discipline effect is not affected by the asymmetry in information. Furthermore, there is another important consequence of the fact that B_t is constant over time. This is the fact that policymakers do not manipulate the public's learning process, neither in the form of signalling their preferences (and hence speeding up the process of learning which would have required a decreasing sequence for the B_ts), nor in the form of slowing down the revision of inflationary expectations (which would have required an increasing sequence for the B_ts). It is worth pointing out that the government is subject to two types of pressures. On one hand, it would like to conceal its preferences in order to slow down the learning. On the other hand, it is eager to behave according to its preferences. In this case, the two forces are perfectly balanced. In other words, the government chooses not to manoeuvre the speed at which private agents glean information. The latter does only depend on the discrepancy between actual inflation and its estimated value, and is therefore greater in the early stages of the game and decrease as the degree of uncertainty of the public is reduced.

The solution to the game when the public's uncertainty tends to zero, converges to the solution of the symmetric information case presented in Section 3.2.

This can be shown by computing the limiting value for the control equation (54) as time tends to infinity. In fact, since μ_{2t} is finite and K_t tends to infinity, as time tends to infinity, as shown in the Appendix B, the third term on the right hand side tends to 0 and equation (54) reduces to equation (20).

The path of the time-consistent inflation rate for a country which switches from a flexible exchange rate regime, where the law of one price always holds, to a pegged exchange rate system, like the ERM, in the presence of asymmetric information, shows an initial decline which exceeds the long run reduction due to the change in the exchange rate regime. It is possible to label this an "overshooting" of the inflation rate owing to the informational asymmetry, or a "honeymoon effect". Immediately after the regime shift, i.e. at time 0, the value of A_0 is:

$$A_0 = 1 - \frac{\bar{C}}{(\lambda+\rho)^2} , \tag{65}$$

and hence inflation equals:

$$\pi_0^p = c\left[1 - \frac{\bar{C}}{(\lambda+\rho)^2}\right] - \frac{h}{(\lambda+\rho)} . \tag{66}$$

The above formula shows that the initial value of A_t is further away from 1 the greater is σ^2_h , because the greater is σ^2_h , the larger $\bar{C}$. This means that the more the public is uncertain about h, the lower the time-consistent inflation rate, so that the overshooting effect is larger. The value of A_0 also depends on ρ and λ the more the government discounts the future payoffs and the more likely a realignment, the closer A_0 is to 1, i.e. the smaller the additional bonus in terms of reduced inflation. Given the initial value of $\pi^P{}_t$, inflation then increases towards the long run level prevailing under the new exchange rate regime, which is lower than the one resulting under the free float system assumed here.

The public's expectations of inflation immediately after the regime switch equal:

$$\pi_0^e = c\left[1 - \frac{\bar{C}}{(\lambda+\rho)^2}\right] - \frac{\bar{h}}{(\lambda+\rho)}\ . \tag{67}$$

It easy to see that the discrepancy between actual and expected inflation, at time 0, depends on the gap between the true value of h and the private agents' prior. The unconditional mean of the distribution from which h has been drawn is not explained in this model, instead it is taken as given. Therefore, it is only possible to say that if the prior is higher than the true value of h, for example because it is based on the past history of high inflation, then the public will over-estimate inflation. Similarly, if the prior is lower than the actual h, maybe because the private sector judges the regime shift as the beginning of a new game, inflation will result in being under-estimated. In expected terms, the public will continue, also after the initial moment after the regime shift, to overpredict the inflation rate, in the first case, and to underpredict in the second case. Although prediction errors disappear when the private sector has learnt the true h, during the learning phase, prediction errors have implications for output. Negative inflationary surprises entail an output loss, whereas positive inflationary surprises boost income. Notice, however, that the path of planned inflation is independent of the prior. In this regard, it is worthwhile pointing out that, if the pegged exchange regime is adopted as an anti-inflationary strategy, whilst in a symmetric information scenario the reduction in the time-consistent inflation rate occurs with no output loss, in an incomplete information setup it is possible that, although the government's policy involves lower inflation, expected inflation remains high. If this occurs, the sacrifice ratio is greater than in the symmetric information framework.

The model presented above can also be used to examine a shift from a free floating regime to a monetary union, setting λ equal to zero.

3.6. Concluding remarks

This chapter analyses the consequences of a shift in the exchange rate regime, from a free float with PPP to a pegged exchange rate regime with realignments, like the ERM (or to a monetary union), on the actual and expected inflation rate. The main feature of the model is the presence of uncertainty and asymmetric information concerning the government's preferences about competitiveness. In this framework, the public's learning and the government's optimal policy are worked out.

The main findings of this chapter are: i) immediately after the regime shift, there is a "honeymoon effect" where inflation drops to a level that is lower than the one which prevails once private agents have learnt the true value of the preference parameter. There is an "overshooting" of planned inflation, due to the authorities' information advantage which allows them to sustain lower time-consistent inflation rates [17]. In the presence of asymmetric information, the advantage of tying one's

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[17] The reduction in planned inflation entails also a reduction in actual inflation in expected terms (where the expectation is done with respect to the government's information set).

hands is temporarily increased. ii) The reduction of actual inflation could be accompanied by output losses for certain values of the public's prior concerning the unknown preference parameter. iii) The policymakers' optimal strategy does not involve either signalling or concealing of the government's preferences. The speed of learning is not affected by the authorities' policy. iv) As time passes and private agents learn, inflation converges to the symmetric information equilibrium which is characterized by lower inflation compared to that of the initial free float regime. Hence, policymakers cannot conceal their preferences indefinitely.

These results, which take fully into account the strategic interactions between the government and the private sector, challenge the view that informational asymmetries and the public's learning could be responsible for the slow decline in inflation, observed in some countries, like France and Italy, subsequent to joining the ERM; previous findings were based only on the expected inflation path. The public's inflationary expectations, however, have an important role to play also in the model developed here, since they determine, together with actual inflation, the level of output. In fact, whilst the government's strategy does not depend on the private sector's prior for the unknown preference parameter, the path of expected inflation does depend on it. The conclusion reached in this chapter is consistent with the experience of the UK after joining the ERM, when inflation declined rapidly, but then rose again; during the decline in inflation high output costs were incurred.

Appendix B

The solution to the differential equation for μ_{1t}.

The differential equation for μ_{1t} is:

$$(\lambda+\rho)\mu_{1t} - \dot{\mu}_{1t} = h \ . \qquad (68)$$

The solution to this linear deterministic differential equation is straightforward:

$$\mu_{1t} = \frac{h}{(\lambda+\rho)} + \left[\mu_{10} - \frac{h}{(\lambda+\rho)}\right] e^{(\lambda+\rho)t} \qquad (69)$$

and the only value for μ_{10} which ensures that μ_{1t} is always finite makes μ_{1t} time-invariant, i.e.:

$$\mu_{1t} = \frac{h}{(\lambda+\rho)}. \qquad (70)$$

The solution to the differential equation for μ_{2t}.

The differential equation for μ_{2t} is as follows:

$$\dot{\mu}_{2t} = \left[(\lambda+\rho) + \frac{B_t^2}{(\lambda+\rho)^2 K_t}\right] \mu_{2t} - \frac{cB_t}{(\lambda+\rho)} . \qquad (71)$$

Recalling that

$$K_t = \frac{1}{(\lambda+\rho)^2}\int_0^t B_s^2 ds + \frac{1}{\sigma_h^2} \tag{72}$$

it is possible to see that:

$$\frac{B_t^2}{(\lambda+\rho)^2 K_t} = \frac{d\ \log K_t}{dt}\ . \tag{73}$$

It follows that the solution to equation (71) is:

$$\mu_{2t} = \frac{e^{(\lambda+\rho)t}\ K_t c}{(\lambda+\rho)}\left[\bar{C} - \int_0^t B_s \frac{e^{-(\lambda+\rho)s}}{K_s} ds\right] \tag{74}$$

where $\bar{C}$ is an arbitrary constant. The choice of $\bar{C}$ is made to ensure that μ_{2t} is finite at all times. There is a unique constant which satisfies this requirement and this is the following:

$$\bar{C} = \lim_{t\to\infty} \int_0^t B_s \frac{e^{-(\lambda+\rho)s}}{K_s} ds\ . \tag{75}$$

It is possible to show, using De L'Hopital rule, that the limiting value for μ_{2t} as time tends to infinity is:

$$\lim_{t\to\infty} \mu_{2t} = \frac{cB_t}{(\lambda+\rho)^2}\ . \tag{76}$$

CHAPTER 4

THE IMPORTANCE OF THE INFORMATION STRUCTURE:

AN EXERCISE BASED ON CHAPTER 3

4.1 Introduction

In this chapter, an exercise is presented which uses the model developed in Chapter 3, Section 3.2. The exercise consists of changing the source of uncertainty facing the private sector. Here, the uncertainty is placed in the government's preference parameter c. Thus, it is assumed that the public does not know the weight attached to inflationary surprises by the government.

The issue examined here is of some interest in the study of policymakers' optimal strategies in the presence of asymmetric information. The rationale for this analysis is to see whether changing the source of uncertainty affects the authorities' optimal policy and, in particular, whether this modifies the government's behaviour towards learning manipulation. The comparison of the solution to be derived in this chapter to that obtained in Chapter 3 will highlight the differences, if any, brought about by different information asymmetries embedded in the same model. The solution procedure employed here mirrors that of the previous chapter.

This chapter is organized as follows. Section 4.2 describes the information structure. In Section 4.3, the learning process is worked out. In Section 4.4, the optimal government policy is derived; the ensuing discussion compares and contrasts the authorities' strategy to that of Chapter 3. Some concluding remarks follow in Section 4.5.

4.2 The information structure of the model

The information structure assumed here differs from that of Chapter 3 in one aspect only: the asymmetry in information between the government and the private sector concerns the preference parameter c instead of h. This means that the public does not know the policymakers' propensity to create inflationary surprises.

The distribution from which c has been drawn by the government, at the beginning of the game is common knowledge, and it is:

$$c \sim N(\bar{c}, \sigma_c^2), \tag{1}$$

where $\bar{c}$ is the private agents' prior, or the unconditional mean of the distribution; σ^2_c is the variance of the distribution and measures the public's degree of uncertainty.

The conjectural function for the government's policy remains unchanged and is reported below for convenience:

$$\pi_t^P = cA_t - \frac{h}{(\lambda+\rho)}B_t . \tag{2}$$

4.3 The public's learning

The private sector works out the best estimate of the parameter c, $\hat{c}_t$, based on the available observations of actual inflation. The public's optimal strategy, which minimizes the mean square error of the inflation forecast, is:

$$\pi_t^e = \hat{c}_t A_t - \frac{h}{(\lambda+\rho)} B_t \; . \tag{3}$$

The filtering problem can now be stated in its standard form:

$$dc = 0 \; ; \qquad E_0\,(c) = \bar{c}, \;\; E_0\,[c - E_0\,(c)]^2 = \sigma_c^2 \; . \tag{4}$$

$$dp_t = \pi_t^p dt + dz_t = \left[cA_t - \frac{h}{(\lambda+\rho)} B_t \right] dt + dz_t \; , \tag{5}$$

where the first equation describes the system, c, and the second describes the evolution of the observations.

The stochastic differential equation for the Kalman-Bucy filter results in being:

$$d\hat{c}_t = \frac{A_t}{K_t} \left[dp_t - \left[\hat{c}_t A_t - \frac{hB_t}{(\lambda+\rho)} \right] dt \right] \; . \tag{6}$$

where K_t is the inverse of the solution to the Riccati equation and equals:

$$K_t = \int_0^t A_s^2 ds + \frac{1}{\sigma_c^2} \; . \tag{7}$$

The public's estimate of the parameter c starts from the prior $\bar{c}$, at time 0, and is revised according to equation (6) thereafter. As time tends to infinity, it can be shown that $\hat{c}_t$ converges to the true value of c.

Notice that here the speed at which private agents revise their beliefs about the unknown parameter, c, depends on two factors. i) The discrepancy between the true and expected value of c. ii) A_t, which is one of the undetermined coefficients to be worked out in the government's optimization. If A_t results in being unity, this means that the policymakers do not affect the speed at which the public updates its estimate. If, instead, A_t results in being different from one, this implies that the government manipulates the learning speed. If the A_ts form a decreasing sequence, the authorities signal their preferences and facilitate the solving of the public's signal extraction problem. Likewise, if the A_ts form an increasing sequence, the government tries to conceal information, hence it slows down the learning process.

4.4 The government's optimization

The government's objective function is the same as in the previous chapter, as is the equation describing the evolution of the first state variable, competitiveness. On the other hand, the equation showing the time path of the second state variable, which now is $\hat{c}_t$, is given by equation (6).

The government's optimization is expressed by the following equations:

$$\max_{\pi_t^p} E_{gov} \left[\int_0^\infty e^{-(\lambda+\rho)t} [c(\pi_t - \pi_t^e) - \frac{1}{2}(\pi_t^p)^2 + hq_t] \, dt \right] , \tag{8}$$

$$dq_t = -\pi_t^p \, dt - dz_t , \tag{9}$$

$$d\hat{c}_t = \frac{A_t}{K_t} \left[\left[\pi_t^p - \left[\hat{c}_t A_t - \frac{hB_t}{(\lambda+\rho)} \right] \right] dt + dz_t \right] , \tag{10}$$

where π^e_t is given by equation (3) and K_t is defined in equation (7). π^p_t is the government's control variable as before.

Proposition

The equilibrium strategy of the government, $(\pi^p_t)^*$, obtained from the game specified above is:

$$(\pi^p_t)^* = c\left[1 + A_t e^{(\lambda+\rho)t}\left[\int_0^t \frac{A_s e^{-(\lambda+\rho)s}}{K_s}ds - \bar{C}\right]\right] - \frac{h}{(\lambda+\rho)} \tag{11}$$

where

$$\bar{C} = \lim_{t\to\infty}\int_0^t \frac{A_s e^{-(\lambda+\rho)s}}{K_s}ds \ . \tag{12}$$

and A_t satisfies the following differential equation

$$A_t\ddot{A}_t - 2(\dot{A}_t)^2 - (\lambda+\rho)A_t^2\dot{A}_t + (\lambda+\rho)^2A_t^2(A_t-1)^2 = 0 \ . \tag{13}$$

Proof

The following value function is conjectured:

$$V_t\,(q_t\,,\hat{c}_t) = e^{(\lambda+\rho)t}\,W_t\ , \tag{14}$$

where

$$W_t = \mu_{0t} + \mu_{1t}\,q_t + \mu_{2t}\,\hat{c}_t\ . \tag{15}$$

The HJB equation results in being:

$$(\lambda+\rho)W_t - \dot{\mu}_{0t} - \dot{\mu}_{1t}\, q_t - \dot{\mu}_{2t}\, \hat{c}_t = \max_{\pi_t^P} c\pi_t^P - c\left[\hat{c}_t A_t - \frac{hB_t}{(\lambda+\rho)}\right]$$

$$- \frac{1}{2}(\pi_t^P)^2 + hq_t - \mu_{1t}\pi_t^P + \frac{\mu_{2t}A_t}{K_t}\left[\pi_t^P - \left[\hat{c}_t A_t - \frac{hB_t}{(\lambda+\rho)}\right]\right] \tag{16}$$

From equation (16), the first order condition yields the following formula for the control

$$(\pi_t^P)^* = c - \mu_{1t} - \frac{\mu_{2t}A_t}{K_t}\,. \tag{17}$$

Equating the terms in q_t on the left hand side and right hand side of the HJB equation, a differential equation in μ_{1t} is obtained; the solution to this is:

$$\mu_{1t} = \frac{h}{(\lambda+\rho)}. \tag{17}$$

Once the terms in c_t are equated in the HJB equation, the resulting deterministic differential equation is:

$$\dot{\mu}_{2t} = \left[(\lambda+\rho) + \frac{A_t^2}{K_t}\right]\mu_{2t} + cA_t\,. \tag{19}$$

The solution to equation (19) is shown below:

$$\mu_{2t} = e^{(\lambda+\rho)t} K_t c \left[\int_0^t \frac{A_s e^{-(\lambda+\rho)s}}{K_s} ds - \bar{C} \right] , \tag{20}$$

where

$$\bar{C} = \lim_{t \to \infty} \int_0^t \frac{A_s e^{-(\lambda+\rho)s}}{K_s} ds . \tag{21}$$

μ_{2t} is always negative and finite. Equation (18) and (20) can now be substituted in the optimal control equation (17). The solution derived is of the form initially postulated, so that the rational expectations equilibrium of the game is obtained. This equilibrium is perfect bayesian.

The coefficients A_t and B_t are:

$$B_t = 1, \tag{21}$$

$$A_t = 1 + A_t e^{(\lambda+\rho)t} \left[\int_0^t A_s \frac{e^{-(\lambda+\rho)s}}{K_s} ds - \bar{C} \right] < 1 . \tag{22}$$

Since equation (23) is not an explicit formula for A_t, it is necessary to rearrange it and then differentiate it with respect to time twice in order to obtain a differential equation which describes the time path of A_t. It turns out to be the same as the one obtained for B_t in Chapter 2, that is:

$$A_t \ddot{A}_t - 2(\dot{A}_t)^2 - (\lambda+\rho)A_t^2 \dot{A}_t + (\lambda+\rho)^2 A_t^2 (A_t - 1)^2 = 0 . \tag{23}$$

This completes the proof. The solution to this second order differential equation has already been worked out in Chapter 2. Hence, only its characteristics are recalled here, in order to compare this solution to that of Chapter 3. The value of A_t at time 0 is:

$$A_0 = \frac{1}{1+C} , \tag{25}$$

therefore A_0 lies in the open interval (0,1). The lower σ^2_c and the greater are ρ and λ, the closer A_0 is to 1. Unity is the only relevant stationary value for A_t, and A_t converges to it as time tends to infinity. The sequence of A_ts starts from A_0, given by equation (25), and then increases towards 1, namely towards the full information solution. The meaning of this solution is that, owing to the information asymmetry, the government chooses a lower time-consistent inflation rate compared to the full information scenario. This is also a feature of the solution obtained in Chapter 3, when the uncertainty was placed in h. However, the crucial difference between the solution obtained here and that of the previous chapter is that here the authorities' optimal policy does involve learning manipulation, whereas this was not the case earlier on. In particular, the government reduces the speed at which learning takes place by choosing the A_ts initially small and then increasingly larger as the public learns. The same result would apply if the uncertainty concerned the weight attached by the government to inflation, namely about a, assuming that the inverse of a was normally distributed.

4.5 Concluding remarks

This exercise highlights that, not only information asymmetries and learning change the solution dramatically compared to the full information case, but also that information asymmetries concerning different parameters, although embedded in the same model, bring about different solutions.

The major difference in the government's optimal policy with respect to Chapter 3 is that when the uncertainty concerns h, i.e. the weight the government attaches to competitiveness, the authorities do not affect the speed at which the public's learning takes place. On the other hand, when the uncertainty regards c, that is the government's propensity to create inflation surprises, as it has been the case in this chapter, the government does embark on learning manipulation which slows down the public's revision of expectations.

In conclusion, the source of uncertainty does matter, as it affects the government's optimal strategy and determines whether learning manipulation is carried out by the policymakers.

CONCLUSION

The thesis expands our understanding of the interactions between the government and the public. Part I idenfies the time-consistency issues that arise in a managed exchange rate regime, while Parts II and III shed some light on the strategic behaviour of the policymakers and the private sector in an environment with uncertainty and incomplete information.

Part I of this thesis deals with optimal exchange rate management and focuses on the time-consistency issues that arise when a country pursues exchange rate stabilization but wants to retain some monetary independence. The analysis conducted in Chapter 1 points out that discrepancies between the time-consistent policy and the optimal (time-inconsistent) linear rule can emerge both in the long run equilibrium and in the path to the steady state. In particular, if there is no conflict between the exchange rate and the money target, then the incentive-compatible policy differs from the optimal one only in the speed of convergence to the long run equilibrium. The optimal linear rule turns out to be faster than the discretionary outcome, as it would pay the government to commit itself to a rule which brings the exchange rate closer to the target. The situation in which there is a conflict of objectives is more interesting and yields differing results even in the long run. If the government pursues a strong exchange rate but loose money target, then the long run money stock is higher in the time-consistent case than with the optimal linear rule. The reverse is true if the government's objectives are tight money and a depreciated exchange rate. Under the time-consistent policy there is a long run bias which can be avoided by precommitment to a linear rule. The speed of convergence also differs as between the time-consistent policy and the optimal linear rule, but which of the two is faster depends on the type of conflict and on the initial condition. The findings of this chapter offer a rationale for the setting up of institutions which can provide the commitment technology in order to make the time-inconsistent policy a sustainable outcome.

Parts II and III shed some light on the consequences of the presence of uncertainty and asymmetric information on optimal policy planning.

Chapter 2 analyses a closed economy model *a' la* Barro-Gordon and the attention is focused on the government's time-consistent policy. The presence of asymmetric information initially permits lower time-consistent inflation rates than in the symmetric information case. But the time path of inflation is increasing and converges to the complete information equilibrium. Furthermore, the government's policy affects the speed at which the private sector learns with the result that the public's revision of expectations is slowed down (although, in the long run, private agents do learn the true preferences of the government). Thus, the information structure is a crucial element in this model; it enhances the scope for strategic interactions between the players, affecting the government's policy and the private sector's expectations in a substantial way. This model can help explain the 'honeymoon period' which policymakers experience immediately after taking office --here this consists of low time-consistent inflation. It also explains how the government may try to take advantage of this situation by hindering the public's learning process, before the true nature of the government eventually becomes known to the private sector.

The open economy model developed in Part III, Chapter 3, examines the consequences of asymmetric information when a shift in the exchange rate regime takes place. In this chapter, the public is uncertain about how much the government cares about international competitiveness, which turns out to simplify the results. Specifically, the optimal government strategy does not involve either signalling or concealing its preferences. These findings challenge the view that informational asymmetries could account for the slow decline in inflation observed in some countries after joining the Exchange Rate Mechanism of the European Monetary System. In particular, the analysis implies that, immediately after the regime change, inflation is lower than in the steady state itself. This decline in inflation may not be costless, however, as it can be accompanied by high output losses if inflationary expectations start from a high level (depending on the values of the public's prior on the government's preferences). These results which take fully into account the strategic interactions between the government and the private sector, are consistent with the experience of the UK after joining the ERM, when inflation fell sharply but rose subsequently.

Finally, Chapter 4 shows how the way in which the information asymmetry is introduced matters for the determination of government policy. When uncertainty concerns, not the level of competitiveness but the propensity to create inflationary surprises, it turns out that, as in Chapter 2, while at variance with Chapter 3, the government now optimally chooses to delay the public's learning. In short, the results previously obtained for a close economy may well arise in the open economy.

The no-conflict case

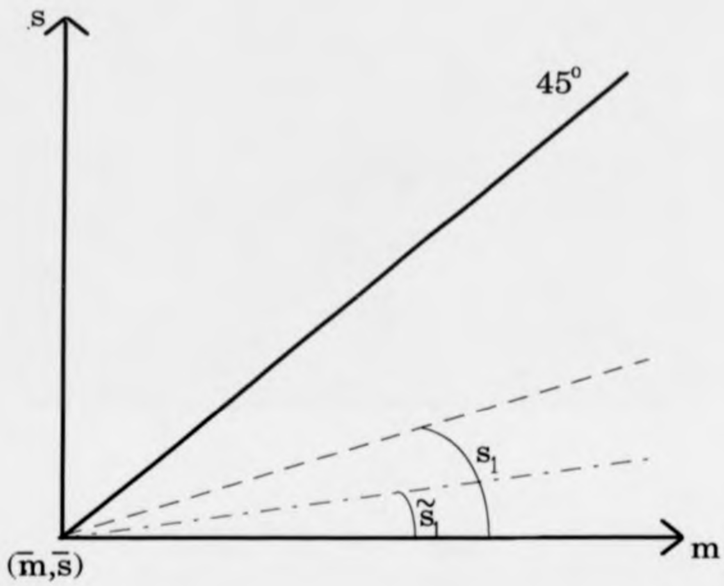

Figure 1

The conflict case

$\bar{s} > \bar{m}$

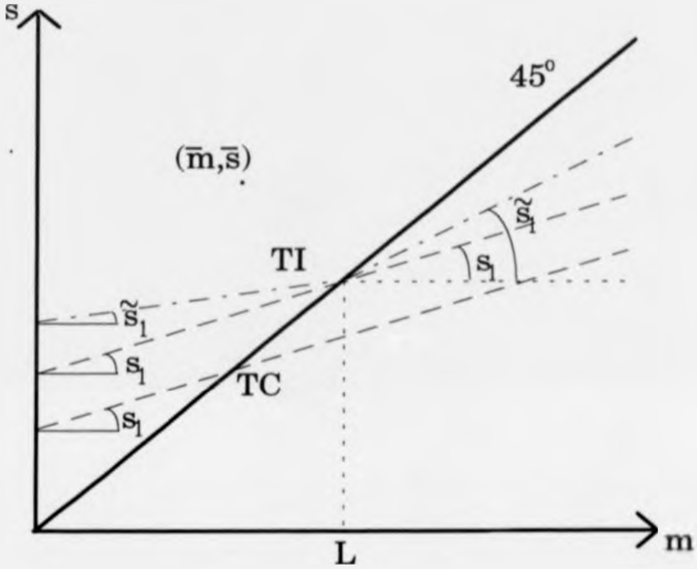

Figure 2

The conflict case

$\bar{s} < \bar{m}$

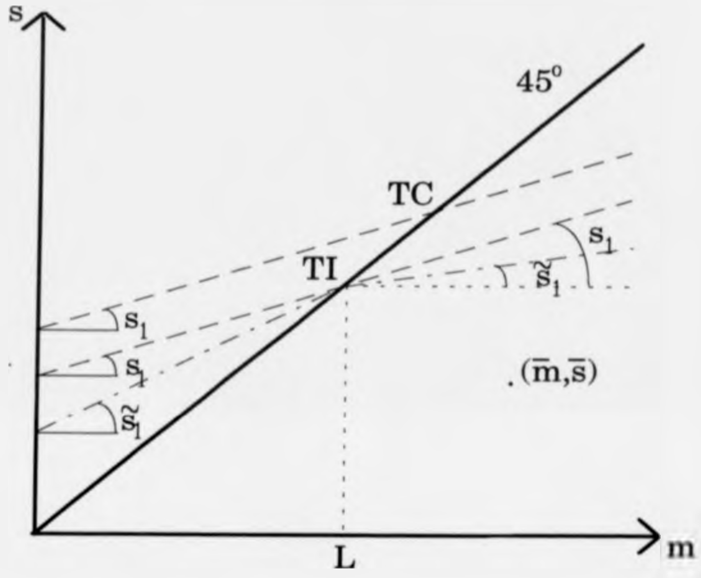

Figure 3

Phase diagram for A

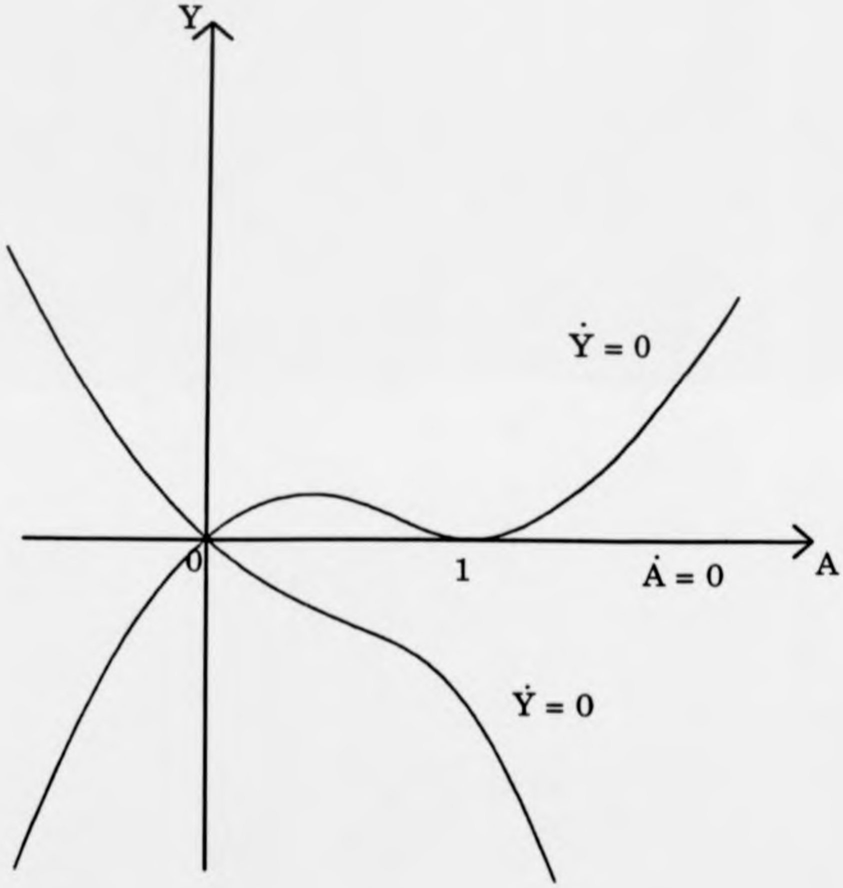

Figure 4

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